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Exploring new features for the (2+1)-dimensional Kundu–Mukherjee–Naskar equation via the techniques of ($G'/G, 1/G$)-expansion and exponential rational function

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Abstract

The aim of the manuscript is to study new optical soliton solutions of the Kundu–Mukherjee–Naskar (KMN) equation via the ($G'/G, 1/G$)-expansion technique and the exponential rational function (ERF) procedure. The results are produced under the constraint conditions, and their graphical representation highlights them. These discoveries might aid in the comprehension of intricate nonlinear phenomena and oceanography. Studying the investigated equation in this paper is very important in explaining oceanographic phenomena such as rogue waves' ocean currents.

Keywords Optical solitons · Exact solutions · Partial differential equation · Kundu–Mukherjee–Naskar equation · Symbolic computation

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1 Introduction

As science has advanced, the exact solution of nonlinear partial differential equations (NPDEs) has gained significance. These solutions are essential for understanding the physical structure that nonlinear models provide when studying NPDEs that arise in various contexts (Arqub 2022; Arqub and Al-Smadi 2020). Condensed material physics, astronomy, nuclear physics, electromagnetic theory, earth science fluid motion, gravity, physical chemistry, elastic medium, energy physics, optic fibers, geophysics, biostatistics, natural physics, chemical mechanics, compound physics, electroch [3] (Arqub 2018, 2020). Among these solutions, optical solutions are crucial for understanding the physical structure that nonlinear models provide when studying nonlinearity that occurs in a variety of contexts.

To deal with the precise solutions of nonlinear differential models, many scientists have put forth a number of various strategies. For example, Arshed et al. dealt with the optical solitons of different equations via the (G'/G^2)-expansion technique (Arshed et al. 2022). Mirzazadeh et al. used the new Kudryashov technique (Mirzazadeh et al. 2021). Osman et al. utilized the generalized unified method to find double-wave solutions for a nonlinear partial differential equation (NPDE) (Osman et al. 2020). Kumar et al. used the sine-Gordon expansion technique. Kumar et al. (2017). Wang et al. used the auxiliary equation method (Wang et al. 2022). Akbulut et al. utilized the modified simple equation (MSE) procedure (Akbulut et al. 2021). Yang and Ma obtained different solutions which are categorized as lump-type solutions (Yang and Ma 2017). Yue et al. employed the improved F-expansion (Yue et al. 2022). Gurefe applied the generalized Kudryashov technique (Gurefe 2020). Kaplan utilized the transformed rational function technique (Kaplan 2018). Raza et al. found auto-B äcklund transformations of some NPDEs (Raza et al. 2022). Tariq et al. applied the Bernoulli sub-ordinary differential equation technique [17].

The $(G'/G, 1/G)$-expansion approach and the exponential rational function procedure are two adaptable methodologies used in this study to create optical solitons of the KMN equation. The following is how this manuscript is arranged: In Sect. 2, we outline the main steps of the $(G'/G, 1/G)$-expansion technique. In Sect. 3, we present the ERF approach. In Sect. 4, we will introduce the governing model. In Sect. 5, we give the utilization of the $(G'/G, 1/G)$-expansion technique to the KMN equation. In Sect. 6, we present the utilization of the exponential rational function procedure to the KMN equation. Finally, we give some conclusions in Sect. 7.

2 The $(G'/G, 1/G)$-expansion technique

The current part of the manuscript will provide an explanation of the $(G\prime/G, 1/G)$-expansion methodology, which is a popular method for generating various types of accurate NPDE solutions. Akbulut et al. (2016); Demiray et al. (2014). First of all, we will take into consideration the following equation:

$$G''(\xi) + \alpha G(\xi) = \iota \tag{1}$$

and choose

$$\Phi = G'/G, \Omega = 1/G, \tag{2}$$

for simplification. If we use Eqs. (1) and (2), we may obtain

$$\Phi' = -\Phi^2 + \imath\Omega - \alpha, \quad \Omega' = -\Phi\Omega. \tag{3}$$

Equation (1) has three different solutions as follows:

1 If $\alpha > 0$, then general solutions of the Eq. (1) will be

$$G(\xi) = m_1 \sin(\sqrt{\alpha}\xi) + m_2 \cos(\sqrt{\alpha}\xi) + \frac{\imath}{\alpha}$$

and we get

$$\Omega^2 = \frac{\alpha}{\alpha^2\psi - \imath^2}(\Phi^2 - 2\imath\Omega + \alpha), \tag{4}$$

where m_1 and m_2 are arbitrary constants and $\psi = m_1^2 + m_2^2$.

2 If $\alpha < 0$, then general solutions of the Eq. (1) will be

$$G(\xi) = m_1 \sinh(\sqrt{-\alpha}\xi) + m_2 \cosh(\sqrt{-\alpha}\xi) + \frac{\imath}{\alpha}$$

and we have

$$\Omega^2 = \frac{-\alpha}{\alpha^2\psi + \imath^2}(\Phi^2 - 2\imath\Omega + \alpha), \tag{5}$$

where m_1 and m_2 are two arbitrary constants and $\psi = m_1^2 - m_2^2$.

3 If $\alpha = 0$, the general solutions of the Eq. (1) will be

$$G(\xi) = \frac{\imath}{2}\xi^2 + m_1\xi + m_2$$

and we have

$$\Omega^2 = \frac{1}{m_1^2 - 2\imath m_2}(\Phi^2 - 2\imath\Omega), \tag{6}$$

where m_1 and m_2 are two arbitrary constants.

We will take into consideration a NPDE with, let's say, x and t as the two independent variables:

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, ...) = 0. \tag{7}$$

The $(G'/G, 1/G)$ expansion technique can be summarized as follows:

1. From the transformation $\xi = x - \omega t$ and with $u(x, t) = u(\xi)$, Eq. (7) may be transformed to an ODE on $u(\xi)$ with

$$P(u, \omega u', u', \omega^2 u'', \omega u'', ...) = 0, \tag{8}$$

where $'$ represents the derivative with respect to ξ.

2. Assume that the solution of Eq. (8) may be shown by a polynomial in Φ and Ω as

$$u(\xi) = \sum_{i=0}^{N} \rho_i \Phi^i + \sum_{i=1}^{N} \varsigma_i \Phi^{i-1}\Omega, \tag{9}$$

where $G = G(\xi)$ is the solution of auxiliary equation Eq. (1), $\rho_i(i = 0, ..., N)$, $\varsigma_i(i = 1, ..., N), \omega, \alpha$ and $\imath$ are constants which will be calculated, and the positive integer

N may be calculated by utilizing the homogeneous balance principle between the highest order derivatives and the nonlinear terms in Eq. (8).

3. When we surrogate Eq. (9) is into Eq. (8), use Eqs. (3) and (4) (here the first case is considered as example) the left-hand side of Eq. (8) can be transformed into a polynomial in Φ and Ω, in which the degree of Ω is not larger than 1. If we equate each coefficient of the polynomial to zero, we find an algebraic equation system in $\rho_i(i=0,\dots,N)$, $\varsigma_i(i=1,\dots,N), \omega, \alpha(\lambda<0), \iota, m_1$ and m_2.

4. Utilize Maple to solve the obtained algebraic solutions. Surrogating the values of $\rho_i(i=0,\dots,N)$, $\varsigma_i(i=1,\dots,N), \omega, \alpha, \iota$, m_1 and m_2 obtained into Eq. (9), it is obtained the travelling wave solutions shown by the hyperbolic functions of Eq. (8).

5. Substitution of Eq. (9) into Eq. (8), utilizing Eqs. (3) and (5) [or Eqs. (3) and (6)] gives the optical soliton solutions of Eq. (8) demonstrated by trigonometric functions (or demonstrated by rational functions).

3 The ERF technique

Suppose the solution of Eq. (8) of the form Ghanbari and Inc (2018); Aksoy et al. (2016):

$$q(\xi)=\sum_{j=0}^{N}\frac{\rho_j}{(1+e^{\xi})^N}, \rho_N\neq 0 \tag{10}$$

where $\rho_j(j=0,1,2,\dots,N)$ are constants will be calculated and N is the balancing number which can be calculated as before. Then, if we substitute Eq. (10) into Eq. (8), we acquire a set of algebraic equations in parameters $\rho_j(j=0,1,2,\dots,N)$ and ω. After that, if we solve the obtained system, we may construct new optical solutions of Eq. (7). This technique is effective, direct and useful for obtaining exact solutions to NPDEs in mathematical physics and optics.

4 Governing Model

In this section, the KMN equation will be examined as follows:

$$iu_t+\lambda_1 u_{xy}+i\lambda_2 u(uu_x^*-u^*u_x)=0 \tag{11}$$

where $u(x,y,t)$ is wave portrait indicated the complex nonlinear wave envelope and $*$ signifies the complex conjugation. The dispersion term in Eq. (11) is identified by the coefficient of λ_1, and the nonlinear term appearing as a coefficient of λ_2 is different from the conventional Kerr law nonlinearity or any known non-Kerr law media.

In order to account for oceanic rogue waves and both hole waves, the KMN equation was originally put forth in 2014. The idea that this model may be used to study optical wave propagation through coherently excited resonant wave guides which are doped with erbium atoms was also put out in 2013. It was mentioned that the model may specifically investigate the phenomenon of light beam bending. Several academics are now streaming results from this model. Previously, it was thought that the model would run into bright and solitary solitons by employing an extended trial function technique. Additionally indicated were higher-order logical solutions (Ekici et al. 2019; Wen 2017). In 2016, rogue wave solutions for the KMN equation have been presented. Following that, various soliton solutions were created using the trial equation methodology and the MSE technique (Qiu et al. 2016; Yildirim 2019a, b). With respect to Kerr law and non-Kerr law nonlinearities, the KMN equation provides an advance version of the nonlinear Schrödinger equation known as current-like nonlinearity.

We will assume the following transformation:

$$u(x, y, t) = q(\xi)e^{i\phi}, \xi = \tau_1 x + \tau_2 y - vt, \tag{12}$$

where $q(\xi)$ and v are the amplitude and velocity of the soliton, and

$$\phi = \gamma_1 x + \gamma_2 y + \omega, \tag{13}$$

here, κ_1 and κ_2 are the soliton frequencies in $x-$ and $y-$directions while ω indicates the soliton wave number.From the substitution of Eqs. (12) and (13) into Eq. (11), yields

$$\lambda_1\tau_1\tau_2 q'' - (\omega + \lambda_1\gamma_1\gamma_2)q + 2\gamma_1\lambda_2 q^3 = 0, \tag{14}$$

from real part

$$v = \lambda_1(\tau_1\gamma_2 + \tau_2\gamma_1). \tag{15}$$

and from imaginary part.

5 Application of the $(G'/G, 1/G)$-expansion technique

The current part of the manuscript presents the utilization of the $(G'/G, 1/G)$-expansion technique to the KMN equation.

According to the homogeneous balancing principle, if we balance the nonlinear term q'' with the highest-order linear term q^3 in Eq. (14), we may find

$$N = 1. \tag{16}$$

Consequently from Eq. (9) we get

$$q(\xi) = \rho_0 + \rho_1\Phi + \varsigma_1\Omega, \tag{17}$$

where ρ_0, ρ_1, ς_1 are constants which will be calculated. Then, we substitute this solution into the reduced equation and collecting the coefficients together and setting them as zero, we find an equation system. There are three cases that need to be explained, as we indicated above:

1. When $\alpha > 0$:

Surrogating Eq. (17) into Eq. (14), utilizing Eqs. (3) and (4), we find a polynomial in Φ and Ω :

$$\Phi^3: \quad -4\lambda_1\tau_1\tau_2\rho_1\alpha^2\psi\iota^2+6\lambda_2\gamma_1\rho_1\varsigma_1^2\alpha\iota^2+2\lambda_1\tau_1\tau_2\rho_1\alpha^4\psi^2+2\lambda_1\tau_1\tau_2\rho_1^4\iota$$
$$+4\lambda_2\gamma_1\rho_1^3\alpha^2\psi\iota^2-2\lambda_2\gamma_1^3\rho_1\alpha^4\psi^2-2\lambda_2\gamma_1\rho_1^3\iota^4-6\lambda_2\gamma_1\rho_1\varsigma_1^2\alpha^3\psi=0,$$

$$\Phi^2\Omega^1: \quad -6\lambda_2\gamma_1\rho_1^2\varsigma_1\iota^4+2\lambda_1\tau_1\tau_2\varsigma_1\lambda^4\psi^2-2\lambda_2\gamma_1\varsigma_1^3\lambda^3\psi-6\lambda_2\gamma_1\rho_1^2\varsigma_1\alpha^4\psi^2$$
$$+2\lambda_2\gamma_1\varsigma_1^3\alpha\iota^2+2\lambda_1\tau_1\tau_2\varsigma_1\iota^4-4\lambda_1\tau_1\tau_2\varsigma_1\alpha^2\psi\iota^2+12\lambda_2\gamma_1\rho_1^2\varsigma_1\alpha^2\psi\iota^2=0,$$

$$\Phi^2\Omega^0: \quad -6\lambda_2\gamma_1\rho_0\varsigma_1^2\lambda^3\psi+12\lambda_2\gamma_1\rho_0\rho_1^2\alpha^2\psi\iota^2-6\lambda_2\gamma_1\rho_0\rho_1^2\lambda^4\psi^2-\lambda_1\tau_1\tau_2\varsigma_1\iota\lambda^3\psi$$
$$+\lambda_1\tau_1\tau_2\varsigma_1\iota^3\lambda-6\lambda_2\gamma_1\rho_0\rho_1^2\iota^4+6\lambda_2\gamma_1\rho_0^2\varsigma_1\lambda\iota^2+4\lambda_2\gamma_1^3\varsigma_1\lambda^2\iota=0$$

$$\Phi^1\Omega^1: \quad 6\lambda_1\tau_1\tau_2\rho_1\iota^3\lambda^2\psi-12\lambda_2\gamma_1\rho_1\varsigma_1^2\alpha\iota^3+12\lambda_2\gamma_1\rho_1^2\varsigma_1\alpha^3\iota\psi-12\lambda_2\gamma_1\rho_0\rho_1\varsigma_1\alpha^4\psi^2$$
$$+24\lambda_2\gamma_1\rho_0\rho_1\varsigma_1\alpha^2\psi\iota^2-3\lambda_1\tau_1\tau_2\rho_1\iota\alpha^4\psi^2-3\lambda_1\tau_1\tau_2\rho_1\iota^5-12\lambda_2\gamma_1\rho_0\rho_1\varsigma_1\iota^4=0$$

$$\Phi^1\Omega^0: \quad -6\lambda_2\gamma_1\rho_0^2\rho_1\iota^4-4\lambda_1\tau_1\tau_2\rho_1\alpha^3\psi^2-\omega\rho_1\iota^4+2\lambda_1\tau_1\tau_2\rho_1\alpha\iota^4+2\omega\rho_1\alpha^2\psi\iota^2-\omega\rho_1\alpha^4\psi^2$$
$$+2\lambda_1\gamma_1\gamma_2\rho_1\alpha^2\psi\iota^2-\lambda_1\gamma_1\gamma_2\rho_1\iota^4-6\lambda_2\gamma_1\rho_1\varsigma_1^2\alpha^4\psi+2\lambda_1\tau_1\tau_2\rho_1\alpha^5\psi^2$$
$$+12\lambda_2\gamma_1\rho_0^2\rho_1\alpha^2\psi\iota^2-\lambda_1\gamma_1\gamma_2\rho_1\alpha^4\psi^2+6\lambda_2\gamma_1\rho_1\varsigma_1^2\alpha^2\iota^2-6\lambda_2\gamma_1^2\rho_0\rho_1\alpha^4\psi^2=0,$$

$$\Phi^0\Omega^1: \quad -2\lambda_2\gamma_1\varsigma_1^3\alpha^3\psi+2\lambda_2\gamma_1\varsigma_1^3\alpha\iota^2-6\lambda_2\gamma_1\rho_1^2\varsigma_1\alpha^4\psi^2+2\lambda_1\tau_1\tau_2\varsigma_1\alpha^4\psi^2$$
$$-6\lambda_2\gamma_1\rho_1^2\varsigma_1^4\iota-4\lambda_1\tau_1\tau_2\varsigma_1\alpha^2\psi\iota^2+2\lambda_1\tau_1\tau_2\varsigma_1\iota^4+12\lambda_2\gamma_1\rho_1^2\varsigma_1\alpha^2\psi\iota^2=0,$$

$$\Phi^0\Omega^0: \quad -\lambda_1\tau_1\tau_2\varsigma_1\iota\alpha^3\psi-6\lambda_2\gamma_1\rho_0\rho_1^2\alpha^4\psi^2-6\lambda_2\gamma_1\rho_0\varsigma_1^2\lambda^3\psi+4\lambda_2\gamma_1\varsigma_1^3\alpha^2\iota$$
$$-6\lambda_2\gamma_1\rho_0\rho_1^2\iota^4+\lambda_1\tau_1\tau_2\varsigma_1\iota^3\alpha+6\lambda_2\gamma_1\rho_0\varsigma_1^2\alpha\iota^2+12\lambda_2\gamma_1\rho_0\rho_1^2\alpha^2\psi\iota^2=0.$$

From the solutions of the algebraic equation system, one finds:

$$\rho_0=0,\rho_1=\pm\frac{\sqrt{\frac{\lambda_1\tau_1\tau_2}{\lambda_2\gamma_1}}}{2},\varsigma_1=\pm\sqrt{\frac{\lambda_1\tau_1\tau_2\iota^2-\lambda_1\tau_1\tau_2\alpha^2\psi}{4\lambda_2\alpha\gamma_1}},\omega=\frac{\lambda_1(\alpha\tau_1\tau_2-2\gamma_1\gamma_2)}{2}, \tag{18}$$

where $\psi=m_1^2+m_2^2$, $\alpha\neq 0$. We may find the optical soliton solution of Eq. (11) by surrogating these values into Eq. (17), using Eqs. (2) and (4).

$$u_1(x,y,t)=\pm e^{i(\gamma_1x+\gamma_2y+\frac{\lambda_1(\alpha\tau_1\tau_2-2\gamma_1\gamma_2)}{2})}\frac{\alpha\left(\sqrt{\frac{\alpha\lambda_1\tau_1\tau_2}{\lambda_2\gamma_1}}\left(m_1\cos\left(\sqrt{\alpha}(\tau_1x+\tau_2y-vt)\right)\right)-\sqrt{\frac{\alpha\lambda_1\tau_1\tau_2}{\lambda_2\gamma_1}}m_2\sin\left(\sqrt{\alpha}(\tau_1x+\tau_2y-vt)\right)+\sqrt{\frac{\lambda_1\tau_1\tau_2(-\iota^2+\alpha^2(m_1^2+m_2^2))}{4\lambda_2\lambda\gamma_1}}\right)}{2\alpha(m_1\sin\left(\sqrt{\alpha}(\tau_1x+\tau_2y-vt\right)+m_2\cos\left(\sqrt{\alpha}(\tau_1x+\tau_2y-vt)\right))+2\iota}, \tag{19}$$

where $v=\lambda_1(\tau_1\gamma_2+\tau_2\gamma_1)$.

Particularly, when $m_1=0$, $m_2>0$ and $\iota=0$ in Eq. (19), we find the following periodic solution:

$$u_{1,1}(x,y,t)=\pm\, e^{i(\gamma_1x+\gamma_2y+\frac{\lambda_1(\alpha\tau_1\tau_2-2\gamma_1\gamma_2)}{2})}\frac{\alpha\left(\sqrt{\frac{\alpha\lambda_1\tau_1\tau_2}{\lambda_2\gamma_1}}-\sqrt{\frac{\alpha\lambda_1\tau_1\tau_2}{\lambda_2\gamma_1}}m_2\sin\left(\sqrt{\alpha}(\tau_1x+\tau_2y-vt)\right)+\sqrt{\frac{\lambda_1\tau_1\tau_2\alpha^2m_2^2}{4\lambda_2\alpha\gamma_1}}\right)}{2\alpha m_2\cos(\sqrt{\alpha}(\tau_1x+\tau_2y-vt))}, \tag{20}$$

where $v=\lambda_1(\tau_1\gamma_2+\tau_2\gamma_1)$ and when $m_2=0$, $m_1>0$ and $\iota=0$, we find the following periodic solution:

$$u_{1,2}(x,y,t) = \pm\, e^{i(\gamma_1 x+\gamma_2 y+\frac{\lambda_1(\alpha\tau_1\tau_2-2\gamma_1\gamma_2)}{2})} \frac{\alpha\left(\sqrt{\frac{\alpha\lambda_1\tau_1\tau_2}{\lambda_2\gamma_1}}\left(m_1\cos\left(\sqrt{\alpha}(\tau_1 x+\tau_2 y-vt)\right)\right)+\sqrt{\frac{\lambda_1\tau_1\tau_2\alpha^2 m_1^2}{4\lambda_2\alpha\gamma_1}}\right)}{2\alpha m_1\sin\left(\sqrt{\alpha}(\tau_1 x+\tau_2 y-vt)\right))}, \tag{21}$$

where $v = \lambda_1(\tau_1\gamma_2 + \tau_2\gamma_1)$ (Fig. 1).

2. When $\alpha < 0$:

We find a polynomial in Φ and Ω by surrogating Eq. (17) into Eq. (14), using Eqs. (3) and (5). Then from the solutions of this system, we find

$$\rho_0 = 0, \rho_1 = \pm\frac{\sqrt{\frac{\lambda_1\tau_1\tau_2}{\lambda_2\gamma_1}}}{2}, \varsigma_1 = \pm\sqrt{-\frac{\lambda_1\tau_1\tau_2\imath^2+\lambda_1\tau_1\tau_2^2\alpha\psi}{4\lambda_2\alpha\gamma_1}}, \omega = \frac{\lambda_1(\alpha\tau_1\tau_2-2\gamma_1\gamma_2)}{2},. \tag{22}$$

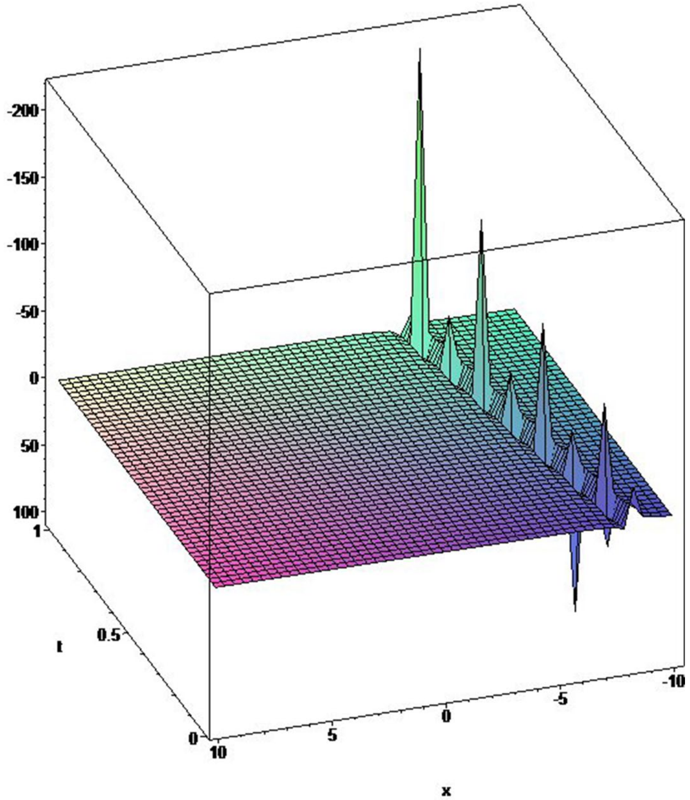

Fig. 1 Graphical representation of $u_1(x,y,t)$ when $\imath = 0, m_1 = 0, \lambda_1 = 0.9, \tau_1 = 0.2, \tau_2 = 0.4, \lambda_2 = 0.8, \alpha = 0.9, m_2 = 1, \gamma_1 = 1.1, \gamma_2 = 1.2, y = 0$

where $\psi = m_1^2 - m_2^2$. We may find the optical soliton solutions of Eq. (11) by surrogating these values into Eq. (17), using Eqs. (2) and (5):

$$u_2(x,y,t) = \pm e^{i(\gamma_1 x+\gamma_2 y+\frac{\lambda_1(\alpha\tau_1\tau_2-2\gamma_1\gamma_2)}{2})} \frac{\alpha\left(\sqrt{-\alpha\frac{\lambda_1\tau_1\tau_2}{\lambda_2\gamma_1}}\left(m_1\cosh\left(\sqrt{-\alpha}(\tau_1 x+\tau_2 y-vt)\right)+m_2\sinh\left(\sqrt{-\alpha}(\tau_1 x+\tau_2 y-vt)\right)\right)+\sqrt{-\frac{\lambda_1\tau_1\tau_2\left(\imath^2+\alpha^2 m_1^2-\alpha^2 m_2^2\right)}{4\lambda_2\alpha\gamma_1}}\right)}{2(m_1(\sinh\left(\sqrt{-\alpha}(\tau_1 x+\tau_2 y-vt)\right)\alpha+m_2\cosh\left(\sqrt{-\alpha}(\tau_1 x+\tau_2 y-vt)\right)\alpha+\imath))}, \tag{23}$$

where $v = \lambda_1(\tau_1\gamma_2 + \tau_2\gamma_1)$.

Particularly, when $m_1 = 0$, $m_2 > 0$ and $\imath = 0$ in Eq. (23), we get the following solitary solution

$$u_{2,1}(x,y,t) = \pm\, e^{i(\gamma_1 x+\gamma_2 y+\frac{\lambda_1(\lambda\tau_1\tau_2-2\gamma_1\gamma_2)}{2})} \frac{\alpha\left(\sqrt{-\alpha\frac{\lambda_1\tau_1\tau_2}{\lambda_2\gamma_1}}\left(m_2\sinh\left(\sqrt{-\alpha}(\tau_1 x+\tau_2 y-vt)\right)\right)+\sqrt{\frac{\lambda_1\tau_1\tau_2\alpha^2 m_2^2}{4\lambda_2\alpha\gamma_1}}\right)}{2m_2\cosh\left(\sqrt{-\alpha}(\tau_1 x+\tau_2 y-vt)\right)\alpha}, \tag{24}$$

and when $m_2 = 0, m_1 > 0$ and $\imath = 0$, we get the following solitary solution

$$u_{2,2}(x,y,t) = \pm\, e^{i(\gamma_1 x+\gamma_2 y+\frac{\lambda_1(\alpha\tau_1\tau_2-2\gamma_1\gamma_2)}{2})} \frac{\alpha\left(\sqrt{-\alpha\frac{\lambda_1\tau_1\tau_2}{\lambda_2\gamma_1}}\left(m_1\cosh\sqrt{-\alpha}(\tau_1 x+\tau_2 y-vt)\right)+\sqrt{-\frac{\lambda_1\tau_1\tau_2\alpha^2 m_1^2}{4\lambda_2\alpha\gamma_1}}\right)}{2m_1\sinh\left(\sqrt{-\alpha}(\tau_1 x+\tau_2 y-vt)\right)\alpha}, \tag{25}$$

where $v = \lambda_1(\tau_1\gamma_2 + \tau_2\gamma_1)$ (Fig. 2).

3. When $\alpha = 0$:

We may construct the optical soliton solutions of Eq. (11) by surrogating these values into Eq. (17), using Eqs. (3) and (6):

$$\rho_0 = 0, \rho_1 = \pm\frac{\sqrt{\frac{\lambda_1\tau_1\tau_2}{\lambda_2\gamma_1}}}{2}, \varsigma_1 = \pm\sqrt{-\frac{2\lambda_1\tau_1\tau_2\imath m_2 - m_1^2\lambda_1\tau_1\tau_2}{4\lambda_2\gamma_1}}, \omega = -\lambda_1\gamma_1\gamma_2. \tag{26}$$

and therefore

$$u_3(x,y,t) = \pm e^{i(\gamma_1 x+\gamma_2 y-\lambda_1\gamma_1\gamma_2)}\left(\pm\frac{\sqrt{\frac{\lambda_1\tau_1\tau_2}{\lambda_2\gamma_1}}\left(\imath(\tau_1 x+\tau_2 y-vt)+m_1\right)+\sqrt{\frac{\lambda_1\tau_1\tau_2\left(m_1^2-2\imath m_2\right)}{\lambda_2\gamma_1}}}{\imath(\tau_1 x+\tau_2 y-vt)^2+2m_1(\tau_1 x+\tau_2 y-vt)+2m_2}\right), \tag{27}$$

where $v = \lambda_1(\tau_1\gamma_2 + \tau_2\gamma_1)$ (Fig. 3).

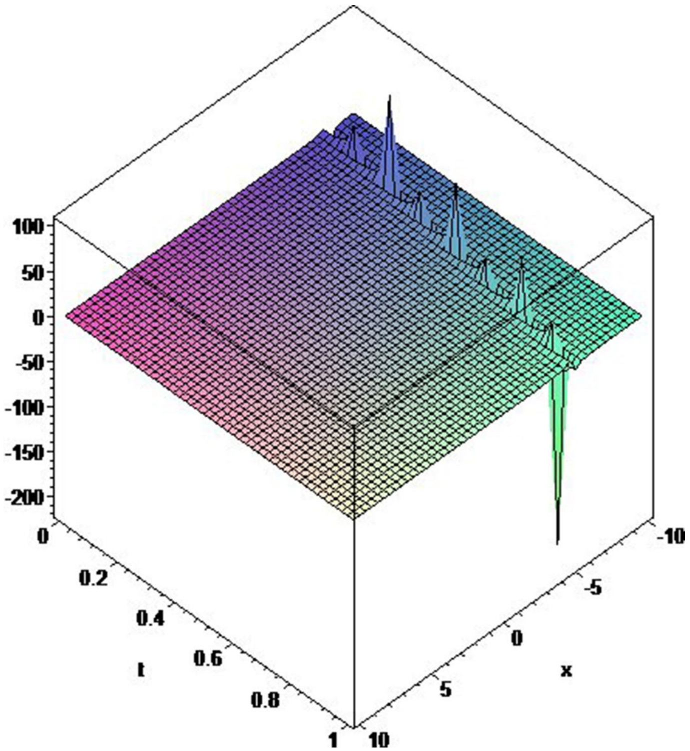

Fig. 2 Graphical representation of $u_2(x,y,t)$ when $\iota=0, m_1=0, \lambda_1=0.9, \tau_1=0.2, \tau_2=0.4, \lambda_2=0.8, \alpha=0.9, m_2=1, \gamma_1=1.1, \gamma_2=1.2, y=0$

6 Application of the ERF technique

Since the balancing number is 1, the desired solution for $q(\xi)$ has the following form:

$$q(\xi)=\rho_0+\frac{\rho_1}{1+e^{\xi}}. \tag{28}$$

Another polynomial in exponential function is obtained by surrogating Eq. (28) into Eq. (14) and combining all terms with the same order of the exponential function's coefficients. Then, by sellecting each coefficient to zero and we find the system below:

$$\begin{aligned}
e^{3\xi} &: -2\lambda_2\gamma_1\rho_0^3-\omega\rho_0-\lambda_1\gamma_1\gamma_2\rho_0=0\\
e^{2\xi} &: \lambda_1\gamma_1\gamma_2\rho_1-3\omega\rho_0-6\lambda_2\gamma_1\rho_0^3-3\lambda_1\gamma_1\gamma_2\rho_0\\
&\quad -6\lambda_2\gamma_1\rho_0^2\rho_1-\lambda_1\gamma_1\gamma_2\rho_1-\omega\rho_1=0,\\
e^{\xi} &: -6\lambda_2\gamma_1\rho_0\rho_1^2-3\omega\rho_0-\lambda_1\tau_1\tau_2\rho_1-3\lambda_1\gamma_1\gamma_2\rho_0\\
&\quad -6\lambda_2\gamma_1\rho_0^3-2\lambda_1\gamma_1\gamma_2\rho_1-12\lambda_2\gamma_1\rho_0^2\rho_1-2\omega\rho_1=0,\\
e^{0\xi} &: -\lambda_1\gamma_1\gamma_2\rho_0-\lambda_1\gamma_1\gamma_2\rho_1-6\lambda_2\gamma_1\rho_0^2\rho_1-\omega\rho_0\\
&\quad -6\lambda_2\gamma_1\rho_0\rho_1^2-2\lambda_2\gamma_1\rho_1^3-2\lambda_2\gamma_1\rho_0^3-\omega\rho_1=0.
\end{aligned}$$

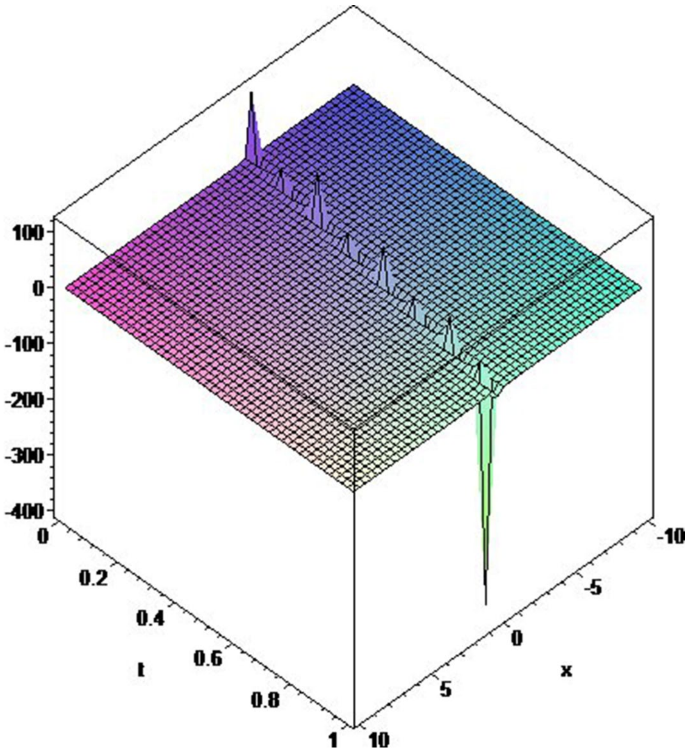

Fig. 3 Graphical representation of $u_3(x, y, t)$ when $\imath = 1, m_1 = 2, \lambda_1 = 0.9, \tau_1 = 0.2, \tau_2 = 0.4, \lambda_2 = 0.8, m_2 = 1, \gamma_1 = 1.1, \gamma_2 = 1.2, y = 0$

Thus, from the substitution of the obtained values in to the solution form, we obtain the following solutions:

$$\rho_0 = \pm \frac{\sqrt{\frac{\lambda_1 \tau_1 \tau_2}{\lambda_2 \gamma_1}}}{2}, \rho_1 = \mp \sqrt{\frac{\lambda_1 \tau_1 \tau_2}{\lambda_2 \gamma_1}}$$

and

$$\omega = -\frac{\alpha(2\gamma_1\gamma_2 + \tau_1\tau_2)}{2}$$

Therefore, the solution takes the form:

$$u_4(x, y, t) = \pm e^{i(\gamma_1 x + \gamma_2 y - \frac{\alpha(2\gamma_1\gamma_2 + \tau_1\tau_2)}{2})} \left(\frac{\lambda_1 \tau_1 \tau_2 \left(\cosh\left(\tau_1 x + \tau_2 y - vt\right) + \sinh\left(\tau_1 x + \tau_2 y - vt\right) - 1\right)}{2\left(\lambda_1 \gamma_1 \sqrt{\frac{\lambda_1 \tau_1 \tau_2}{\lambda_2 \gamma_1}} \left(\cosh\left(\tau_1 x + \tau_2 y - vt\right) + \sinh\left(\tau_1 x + \tau_2 y - vt\right) + 1\right)\right)} \right),$$

where $v = \lambda_1(\tau_1\gamma_2 + \tau_2\gamma_1)$ (Fig. 4).

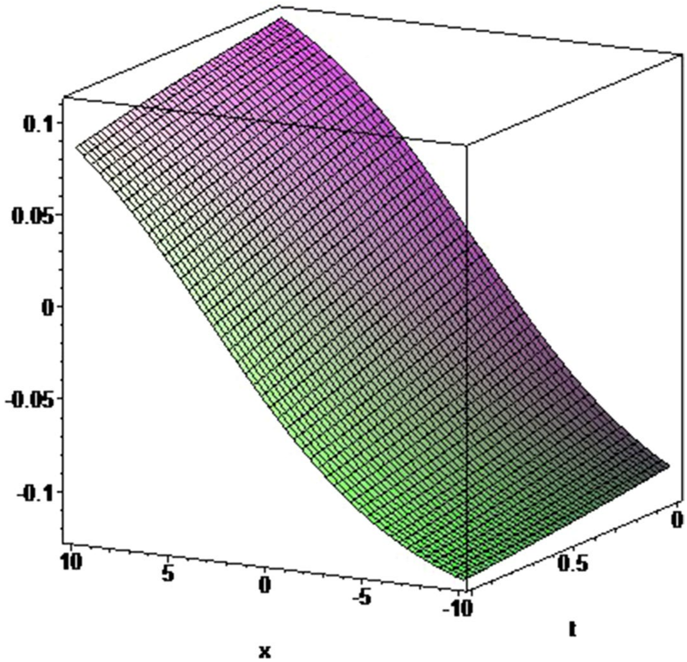

Fig. 4 Graphical representation of $u_4(x,y,t)$ when $\lambda_1 = 0.9, \tau_1 = 0.2, \tau_2 = 0.4, \lambda_2 = 0.8,\ \lambda = 0.9, \gamma_1 = 1.1, \gamma_2 = 1.2, y = 0$

7 Conclusion

The paper has addressed the $(G'/G, 1/G)$-expansion technique and the ERF principle to uncover fairly interesting optical solitons of the KMN equation. Any optical field that does not change throughout propagation is referred to as an optical soliton due to a careful balance between linear and nonlinear effects in the medium. For that reason, optical solitons are important in transmission. For that reason, optical solitons have a great impact on transmission. The obtained results has revealed that the adopted procedure is effective, direct and useful for addressing other NPDEs in mathematical physics. The reportable solutions are novel and gift a beneficial contribution to the literature in nonlinear dynamics. We have used the symbolic computation programs to reach our goal. The results are also produced under the constraint conditions, and their graphical representation highlights them.

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Declarations

Conflict of interest The authors declare that they have no competing interests.

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