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A new exploration of some explicit soliton solutions of q-deformed Sinh-Gordon equation utilizing two novel techniques

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Abstract
In the present work, the q-deformed Sinh-Gordon equation considered by performing new extended generalized Kudryashov method and improved $\tan(\frac{\Phi}{2})$-expansion method. Thanks to these methods, we can effectively obtain rational, hyperbolic and trigonometric solutions by specially choosing the parameters for the existence of solutions. The proposed equation expands the possibilities for modeling physical systems in which symmetry is broken. The obtained solutions are graphically illustrated.

Keywords q-deformed Sinh-Gordon equation · Soliton · New extended generalized Kudryashov method · Improved $\tan(\frac{\Phi}{2})$-expansion method

1 Introduction

Over the past few decades, the exploration of nonlinear wave propagation on the surface of the ocean has piqued the interest of scientists. Nonlinear wave phenomena have been encountered in numerous domains, such as tsunami waves, chemical physics, control theory, and plasma physics etc. (Li and Sun 2020; Zhao et al. 2021; Wang et al. 2021). Analytical and computational soliton solutions can explicitly define these phenomena. Solitons are of great importance as they have new application areas in many fields of positive sciences (Bucket and Thauer 2018; Jabin and Wang 2018). In general, they can be identified by a balance of dispersion and nonlinearity and are produced by various notable nonlinear PDEs such as the Kadomtsev–Petviashvili equation, the nonlinear Schrodinger equation, the Sin-Gordon, Korteweg–de Vries equation, and the Sinh-Gordon equation (Wazwaz and Tantawy 2016; Chabchoub et al. 2015; Wazwaz 2006; Bulut et al. 2016). Indeed, solitons are one of the most distinct solutions of nonlinear dynamics.

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Using soliton solutions, mathematicians and physicists have collaborated to develop a number of techniques for directly examining nonlinear evolution equations (NLEEs). Representing diverse physical phenomena, they have played a key role in a wide range of applications over the last several decades, including nonlinear optics, fluid mechanics, water waves, solid-state physics, elastic media and acoustic waves in crystals (Vakhnenko and Parkes 2016; Islam et al. 2015; Khan and Akbar 2014). Over the last 2 decades, a huge amount of work has been devoted to developing solid and reliable analytical methods for solving these equations. Numerous plans for extracting exact and numerical solutions for these models have been concocted to give adequate data for understanding actual events happening in various positive science fields, for example, the unified technique, to find the exact solitary wave solution, Wronskian formulation, Hirota bilinear method, linear superposition principle, the Painleve approach, invariant subspaces, inverse scattering method, symmetry reduction strategy, and novel auxiliary equation strategy (Raza and Yasmeen 2021; Raza et al. 2021a, b, c; Bagheri and Khani 2020; Sousa and Oliveira 2018; Abdalla et al. 2013; Fabian et al. 2009; Biswas et al. 2018). Huge headway has been made recently, various efficient and proficient techniques for acquiring exact solutions to NLEEs have been laid out.

In the present work we extend the generalized q-deformed Sinh-Gordon equation (Eleuch equation) (Eleuch 2018), given as

$$\frac{\partial^2 m}{\partial x^2} - \frac{\partial^2 m}{\partial t^2} = \left[\sinh_q(m^\gamma)\right]^p - \delta, \tag{1}$$

where $\sinh_q$ is the function of Arai q-deformed defined by

$$\sinh_q(x) = \frac{e^x - qe^{-x}}{2}, \quad 0 < q \leq 1. \tag{2}$$

For $q = 1$ gives standard sinh functions. The $\cosh_q(x)$, $\tanh_q(x)$ and their reciprocal along with their essential characteristics are well described in Eleuch (2018) and Arshed et al. (2019). The proposed approach will be used to find optical soliton solutions of Eq. (1). Contrarily, the current work suggests one of the kind optical solitons and different kinds of solutions to this nonlinear model. This article It shows great progress and continuous improvement over previous work.

The new extended generalized Kudryashov (NEGK) and improved $\tan(\frac{\Phi}{2})$-expansion (ITE) techniques are two of the most widely used approaches for solving nonlinear partial differential equations (NLPDEs) (Seadawy et al. 2021; Özkan and Yasar 2020). These approaches are easy to use and can be used to extract solitary wave solutions from a wide range of NLPDEs. These strategies are beneficial because they can be used to resolve problems with big balance numbers. The NEGK and ITE techniques have been used in this paper to investigate the q-deformed Sinh-Gordon model (Eleuch 2018). Singular soliton solutions, hyperbolic soliton solutions and rational function solutions are among the explicit solutions that can be obtained using the proposed approaches.

The rest of the article is organized as follows. Section 2 covers the basics of mathematical investigation. Section 3 presents an outline of the applied strategies. Section 4 illustrates how to produce soliton solution utilizing the proposed strategy. Section 5 contains a graphical representations of the obtained results for the problems. In the last Sect. 6, the results of this work are presented.

2 The mathematical analysis

The following transformation is used to determine the traveling wave solution of Eq. (1):

$$\zeta = \frac{x - \alpha t}{\sqrt{1 - (\alpha)^2}}, \tag{3}$$

where α denotes the speed of traveling wave. Using Eq. (3), Eq. (1) has been converted into following ODE,

$$\frac{d^2 m(\zeta)}{d\zeta^2} = \left[\sinh_q(m^\gamma(\zeta))\right]^p - \delta. \tag{4}$$

In this paper, Eq. (4) will be studied for $p = 1$, $\gamma = 1$ and $\delta = 0$. The traveling wave solutions for Eq. (4) can be obtained in the subsequent sections using proposed analytical methods by applying the transformation

$$n(\zeta) = e^{m(\zeta)}. \tag{5}$$

After utilizing Eq. (5), Eq. (4) becomes

$$-2n'^2 + 2nn'' - n^3 + qn = 0. \tag{6}$$

3 Outline of the analytical techniques

This section goes over the analytical techniques in detail.

3.1 Method-I: key points of NEGK method

To understand the approach, consider the following nonlinear partial differential equation:

$$M(m, m_x, m_t, m_y, m_{xx}, m_{yy}, m_{tt}, m_{xt}, ...) = 0, \tag{7}$$

where M is the polynomial having nonlinear components of higher order partial derivatives in $m(x, y, t)$. Using the NEGK approach, finding the solution of the generalized q-deformed Sinh-Gordon problem by following the procedures below:

Step1. The process of transforming the traveling wave is

$$m(x, y, t) = \ln(n(\zeta)),\ \zeta = \frac{x + y - \alpha t}{\sqrt{1 - (\alpha)^2}}, \tag{8}$$

where α are nonzero constants. Using the aforementioned transformation equation, Eq. (7) is turned into a nonlinear ordinary differential equation (ODE).

$$L(n, n', n'', n''', ...) = 0, \tag{9}$$

where prime ($'$) denotes the derivative with respect to ζ.

Step2. Assume that there is a rational form solution of Eq. (6),

$$n(\zeta) = \frac{\sum_{n=0}^{r_1} W^n(\zeta)\psi_n}{\sum_{p=0}^{g} W^p(\zeta)\xi_p} = \frac{H[W(\zeta)]}{G[W(\zeta)]}, \tag{10}$$

where $H[W(\zeta)] = \sum_{n=0}^{r_1} W^n(\zeta)\psi_n$ and $G[W(\zeta)] = \sum_{p=0}^{g} W^p(\zeta)\xi_p$ such that ψ_{r_1} and ξ_g cannot be zero,

$$W(\zeta) = \left[\frac{1}{1 \pm a_1^{(s_1\zeta)}}\right]^{\frac{1}{s_1}}, \tag{11}$$

Here $\exp_{a_1}(s_1\zeta) = a_1^{(s_1\zeta)}$, s belongs to Z^+ and W satisfies the expression mentioned below,

$$W' = [W^{s_1+1}(\zeta) - W(\zeta)]\ln(a_1)\,,\ 0 < a_1 \neq 1. \tag{12}$$

Eq. (9) and Eq. (11) implies:

$$\begin{aligned} n'(\zeta) &= W(W^{s_1} - 1)\left[\frac{H'G - HG'}{G^2}\right]\ln(a_1)\,, \\ n''(\zeta) &= W(W^{s_1} - 1)[(s_1+1)W^{s_1} - 1]\left[\frac{H'GF - HG'}{G^2}\right]\ln^2(a_1) \\ &\quad + W^2(W^{s^1} - 1)^2[G(H''G - HG'') - 2H'G'G + 2H(G')^2]\ln^2(a_1). \end{aligned} \tag{13}$$

and so on.

Step3. The homogeneous balancing strategy is used to find the positive integer values of q_1, r_1 in Eq. (10).

If

$$\begin{aligned} &D(n) = r_1 - g,\ D(n') = r_1 - g + s_1,\ D(n'') = r_1 - g + 2s_1,\ \textit{implies that}\ D[n^\kappa n^{(\kappa_1)}] \\ &\quad = (r - g)(\kappa + 1) + s_1\kappa_1. \end{aligned} \tag{14}$$

where D denotes the homogeneous balance of functions.

Step4. In Eq. (9), the equations [(10), (11), (14)] are replaced. A collection of polynomials is created by comparing the comparable powers of $W^n(n = 0, 1, 2, ..)$ and equating these equations to zero, which can then be solved to determine $\psi_n(n = 0, 1, 2, 3, ..., r_1)$, $\xi_p(p = 0, 1, 2, 3, ..., q_1)$, κ, κ_1, and α. As a result, we will be able to come up with precise answers.

3.2 Method-II: key points of ITE technique

This section includes a concise overview of the improved $\tan(\frac{\Phi(\zeta)}{2})$-expansion approach.

Step1. Consider the following NLPDE:

$$M(m, m_x, m_t, m_y, m_{xx}, m_{yy}, m_{tt}, m_{xt}, ...) = 0, \tag{15}$$

M is the polynomial in $m(x, y, t)$ that has higher order partial derivatives with nonlinear components. Above Eq. (15), the traveling transformation from Eq. (8) simplifies Eq. (15) to a nonlinear ODE as follows:

$$L(n, n^{'}, n^{''}, n^{'''}, ...) = 0, \tag{16}$$

Step2. Take Eq. (16), which has a solution of the form:

$$n(\zeta) = \sum_{i=0}^{N} A_i \left[m + \tan\left(\frac{\Phi(\zeta)}{2}\right)\right]^{i} + \sum_{i=1}^{N} B_i \left[m + \tan\left(\frac{\Phi(\zeta)}{2}\right)\right]^{-i}, \tag{17}$$

Here $A_i \neq 0$, $B_i \neq 0$ and $\Phi(\zeta)$ satisfies the ODEs given below:

$$\Phi^{'}(\zeta) = u\ \sin(\Phi(\zeta)) + v\ \cos(\Phi(\zeta)) + w, \tag{18}$$

The following appropriate solutions of (18) will be considered:

Family 1: If $\beta = u^2 + v^2 - w^2 > 0$ and $v \neq w$, then

$$\Phi(\zeta) = 2\arctan\left[\frac{u}{v-w} + \frac{\sqrt{\beta}}{v-w}\tan\left(\frac{\sqrt{\beta}}{2}\hat{\zeta}\right)\right]. \tag{19}$$

Family 2: If $\beta = u^2 + v^2 - w^2 < 0$ and $v \neq w$, then

$$\Phi(\zeta) = 2\arctan\left[\frac{u}{v-w} + \frac{\sqrt{-\beta}}{v-w}\tan\left(\frac{\sqrt{-\beta}}{2}\hat{\zeta}\right)\right]. \tag{20}$$

Family 3: If $\beta = u^2 + v^2 - w^2 > 0$ and $v \neq w$ and u=0, then

$$\Phi(\zeta) = 2\arctan\left[\sqrt{\frac{v+w}{v-w}}\tanh\left(\frac{\sqrt{v^2-w^2}}{2}\hat{\zeta}\right)\right]. \tag{21}$$

Family 4: If $\beta = u^2 + v^2 - w^2 > 0$ and $v \neq 0$ and w=0, then

$$\Phi(\zeta) = 2\arctan\left[\frac{u}{v} + \frac{\sqrt{u^2+v^2}}{v}\tanh\left(\frac{\sqrt{u^2+v^2}}{2}\hat{\zeta}\right)\right]. \tag{22}$$

Family 5: If $v = 0$ and $w = 0$, then

$$\Phi(\zeta) = \arctan\left[\frac{2\exp(u\hat{\zeta}) - 1}{\exp(2u\hat{\zeta}) + 1}\right]. \tag{23}$$

Family 6: If $u = 0$ and $w = 0$, then

$$\Phi(\zeta) = \arctan\left[\frac{\exp(2v\hat{\zeta}) - 1}{\exp(2v\hat{\zeta}) + 1}\right]. \tag{24}$$

Family 7: If $u^2 + v^2 - w^2 < 0$,$w \neq 0$and $v = 0$, then

$$\Phi(\zeta) = 2\arctan\left[-\frac{u}{w} + \frac{\sqrt{-u^2+w^2}}{w}\tanh\left(\frac{\sqrt{-u^2+w^2}}{2}\hat{\zeta}\right)\right]. \tag{25}$$

Family 8: If $w = u$, then

$$\Phi(\zeta) = -2\arctan\left[\frac{(u+v)\exp(v\hat{\zeta})-1}{(u-v)\exp(v\hat{\zeta})-1}\right]. \tag{26}$$

Family 9: If $w=-u$, then

$$\Phi(\zeta) = 2\arctan\left[\frac{\exp(v\hat{\zeta})+v-w}{\exp(v\hat{\zeta})-v-u}\right]. \tag{27}$$

Family 10: if $u=v=w=ku$, then

$$\Phi(\zeta) = 2\arctan[\exp(ku\hat{\zeta})-1]. \tag{28}$$

Family 11: if $u=w=ku$ and $v=-ku$, then

$$\Phi(\zeta) = -2\arctan\left[\frac{exp(ku\hat{\zeta})}{-1+exp(ku\hat{\zeta})}\right]. \tag{29}$$

Family 12; If $u^2+v^2=w^2$, then

$$\Phi(\zeta) = -2\arctan\left[\frac{(u\hat{\zeta}+2)(v+w)}{u^2\hat{\zeta}}\right]. \tag{30}$$

Family 13; If $v=-w$, then

$$\Phi(\zeta) = -2\arctan\left[\frac{u\,exp(u\hat{\zeta})}{w\,exp(u\hat{\zeta})-1}\right]. \tag{31}$$

Family 14; If $v=w$, $u=0$, then

$$\Phi(\zeta) = 2\arctan[w\hat{\zeta}]. \tag{32}$$

Family 15; If $v=-w$, $u=0$ then

$$\Phi(\zeta) = -2\arctan\left[\frac{1}{w\hat{\zeta}}\right]. \tag{33}$$

Family 16; If $u=w$ and $v=0$, then

$$\Phi(\zeta) = -2\arctan\left[\frac{w\hat{\zeta}+2}{w\hat{\zeta}}\right]. \tag{34}$$

Family 17; If $u=0$ and $v=0$, then

$$\Phi(\zeta) = w\hat{\zeta}. \tag{35}$$

Family 18: If $u=w$, then

$$\Phi(\zeta) = 2\arctan\left[\frac{(w+v)\exp(v\hat{\zeta})+1}{(-w+v)\exp(v\hat{\zeta})+1}\right]. \tag{36}$$

where $\hat{\zeta} = \zeta + K$, $A_i(i = 0, 1, 2..., N)$, $B_i(i = 1, 2, 3, ..., N)$, u, v, w are the constants that need to be calculated. Consider the homogeneous balance between the nonlinear term and the highest order derivative term in Eq. (16) to get the value of N in Eq. (17).

Step 3 We derive the equal powers of $\tan(\frac{\Phi(\zeta)}{2})$ and $\cot(\frac{\Phi(\zeta)}{2})$ by using the value of N and entering Eq. (17) into (16), and then we build a system of algebraic equations by setting each collected coefficient to zero.

Step 4 Finding values for $A_0, A_1, ..., A_N, B_1, B_2, ..., B_N$, m *and* α by solving an algebraic system of equations. The answer is obtained after entering these values into Eq. (17).

4 Construction of solutions using proposed methods

This section gives the extraction of soliton solutions for the proposed model Eq. (1) by employing ones of the most efficient analytical methods namely NEGK method and ITE method.

4.1 Method-I

Using the NEGK technique on Eq. (1). Consider the following transformation:

$$m(x, t) = \ln(n(\zeta)),\ \zeta = \frac{x - \alpha t}{\sqrt{1 - (\alpha)^2}}, \tag{37}$$

The real-valued function is $n(\zeta)$, and the traveling wave is α, which indicates the group velocity of the wave packets. Applying transformation Eqs. (2) to Eq. (1), ODE is obtained.

$$-2n'^2 + 2nn'' - n^3 + qn = 0. \tag{38}$$

The homogeneous balancing rule to Eq. (38) yields the values of r, g and s for the purpose of deriving the soliton solution of Eq. (1), which is done by balancing the nonlinear and highest-order derivative term.

$$(r_1 - g)(1 + 1) + 2s_1 = 3(r_1 - g)\ \ implies\ \ r_1 = g + 2s_1. \tag{39}$$

Consider the cases:

Case-1. If we take $g = 0, s_1 = 1,\ r_1 = 2$, Eq. (39) takes the form

$$n(\zeta) = \frac{\psi_0 + \psi_1 W(\zeta) + \psi_2 W^2(\zeta)}{\xi_0}. \tag{40}$$

where ψ_0, ψ_1, ψ_2, and ξ_0 are real constants. $\psi_2 \neq 0$ and $\xi_0 \neq 0$ are also useful. Eq. (40) and Eq. (12) are substituted into Eq. (38), and the identical powers of W^n are set to zero. This is the list of polynomials that were evaluated in order to arrive at the results using Maple programme.

Result 1:

$$q = 0,\ \psi_0 = 0,\ \psi_1 = -4\ln(a_1)^2\xi_0,\ \psi_2 = 4\ln(a_1)^2\xi_0. \tag{41}$$

Utilizing equations Eqs. (41), (37) and (11), we get the solution of Eq. (1) as

$$m(\zeta) = \ln\left[-4\ln^2(a_1)\left(\frac{1}{1 \pm a_1^{\zeta}}\right) + 4\ln^2(a_1)\left(\frac{1}{1 \pm a_1^{\zeta}}\right)^2\right]. \tag{42}$$

Result 2:

$$q = \ln(a_1)^4,\ \psi_0 = \ln(a_1)^2\xi_0,\ \psi_1 = -4\ln(a_1)^2\xi_0,\ \psi_2 = 4\ln(a_1)^2\xi_0. \tag{43}$$

Utilizing equations Eqs. (43), (37) and (11), we get the solution of Eq. (1) as

$$m(\zeta) = \ln\left[\ln^2(a_1) - 4\ln^2(a_1)\left(\frac{1}{1 \pm a_1^{\zeta}}\right) + 4\ln^2(a_1)\left(\frac{1}{1 \pm a_1^{\zeta}}\right)^2\right]. \tag{44}$$

Case-2. If we take $g = 1,\ s_1 = 1,\ r_1 = 3$, Eq. (7) takes the form

$$n(\zeta) = \frac{\psi_0 + \psi_1 W(\zeta) + \psi_2 W^2(\zeta) + \psi_3 W^3(\zeta)}{\xi_0 + \xi_1 W(\zeta)}. \tag{45}$$

where $\xi_0, \xi_1, \beta_2, \psi_0, \psi_1, \psi_2, \psi_3$, are real constants. Also $\xi_1 \neq 0$ and $\psi_3 \neq 0$.

Result 1:

$$q = 0,\ \psi_0 = 0,\ \psi_1 = -4\ln(a_1)^2\xi_0,\ \psi_2 = 4\ln(a_1)^2\xi_0 - 4\ln(a_1)^2\xi_0,\ \psi_3 = 4\ln(a_1)^2\xi_1. \tag{46}$$

Utilizing Eqs. (46), (45) and (11), we get the solution of Eq. (1) as

$$m(\zeta) = \ln\left[\frac{-3\ln^2(a_1)\left(\frac{1}{1\pm a_1^{\zeta}}\right) + 4\ln^2(a_1)\left(\frac{1}{1\pm a_1^{\zeta}}\right)^3}{1 + \frac{1}{1\pm a_1^{\zeta}}}\right]. \tag{47}$$

Result 2:

$$q = 0,\ \psi_0 = 0,\ \psi_1 = -4\ln(a_1)^2\xi_0,\ \psi_2 = 4\ln(a_1)^2\xi_0 - 4\ln(a_1)^2\xi_0,\ \psi_3 = 4\ln(a_1)^2\xi_0. \tag{48}$$

Utilizing Eq. (48), Eq. (45) and Eq. (11), we get the solution of Eq. (1) as

$$m(\zeta) = \ln\left[\frac{\ln^2(a_1) - 3\ln^2(a_1)\left(\frac{1}{1\pm a_1^{\zeta}}\right) + 4\ln^2(a_1)\left(\frac{1}{1\pm a_1^{\zeta}}\right)^3}{1 + \frac{1}{1\pm a_1^{\zeta}}}\right]. \tag{49}$$

4.2 Method-II

In this part, we will solve the generalized q-deformed Sinh-Gordon equation in Eq. (1) using the ITE technique. To get the value of N, first convert it to an ODE using the traveling wave transformation and the homogeneous balance method. The balancing rule on n^3 and nn'' yields the following value of N from Eqs. (2) and (3):

$$N + N + 2 = 3N \ \ implies \ \ N = 2\,. \tag{50}$$

From ITEM, in Eq. (17) setting $m = 0$ the traveling wave solution is of the form,

$$n(\zeta) = \sum_{i=0}^{2} A_i \left[\tan\left(\frac{\Phi(\zeta)}{2}\right)\right]^i + \sum_{i=1}^{2} B_i \left[m + \tan\left(\frac{\Phi(\zeta)}{2}\right)\right]^{-i}, \tag{51}$$

implies

$$\begin{aligned} n(\zeta) =& A_0 \ + \ A_1 \left[\tan\left(\frac{\Phi(\zeta)}{2}\right)\right] + A_2 \left[\tan\left(\frac{\Phi(\zeta)}{2}\right)\right]^2 \\ &+ \ B_1 \left[\tan\left(\frac{\Phi(\zeta)}{2}\right)\right]^{-1} \ + \ B_2 \left[\tan\left(\frac{\Phi(\zeta)}{2}\right)\right]^{-2}. \end{aligned} \tag{52}$$

An algebraic system of equations is obtained by plugging Eq. (52) into (38) and then collecting the like powers of $\tan(\frac{\Phi(\zeta)}{2})$. The following values are obtained by solving the system with Maple: **Set 1:** $A_0 = u^2,\ A_1 = -2uv + 2uw,\ A_2 = v^2 - 2vw + w^2,\ B_1 = 0,\ B_2 = 0.$

5 Results and graphical representation

When values are swapped into the above-mentioned solution families, the algorithmic method is completed. As a result, the solution to the generalized q-deformed Sinh-Gordon equation, as well as their 3D images, are included in this section. The data in set 1 may be used to represent the inventive precise solution of $m(\zeta)$.

$$\begin{aligned} m_1(\zeta) = \ln\Bigg[& 1 + 2\,\frac{u}{v-w} - 2\,\frac{\sqrt{u^2+v^2-w^2}\tan\left(\frac{1}{2}(K+\zeta)\sqrt{u^2+v^2-w^2}\right)}{v-w} \\ & + \left(\frac{u}{v-w} - \frac{\sqrt{u^2+v^2-w^2}\tan\left(\frac{1}{2}(K+\zeta)\sqrt{u^2+v^2-w^2}\right)}{v-w}\right)^2\Bigg]. \end{aligned} \tag{53}$$

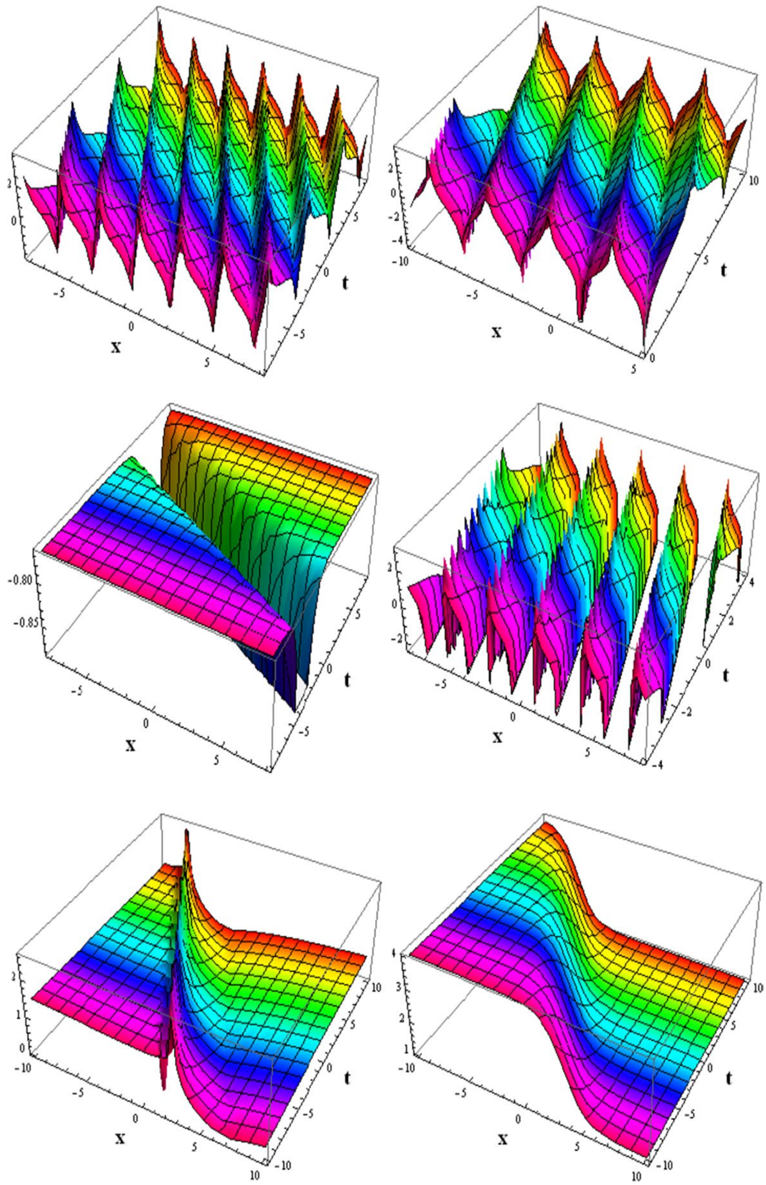

Fig. 1 3D Plot of Eqs. (53)–(58) for $\alpha = -0.5$, $K = 3$, respectively

$$m_2(\zeta) = \ln\left[1 + 2\,\frac{u}{v-w} - 2\,\frac{\sqrt{-u^2-v^2+w^2}\tan\left(\frac{1}{2}(K+\zeta)\sqrt{-u^2-v^2+w^2}\right)}{v-w} + \left(\frac{u}{v-w} - \frac{\sqrt{-u^2-v^2+w^2}\tan\left(\frac{1}{2}(K+\zeta)\sqrt{-u^2-v^2+w^2}\right)}{v-w}\right)^2\right]. \tag{54}$$

$$m_6(\zeta) = \ln\left[2\tan\left(1/2\,\arctan\left(\frac{e^{2v(K+\zeta)}-1}{e^{2v(K+\zeta)}+1}\right)\right) + \left(\tan\left(\frac{1}{2}\,\arctan\left(\frac{e^{2v(K+\zeta)}-1}{e^{2v(K+\zeta)}+1}\right)\right)\right)^2\right]. \tag{55}$$

$$m_7(\zeta) = \ln\left[1 - 4\,\frac{u}{w} + 4\,\frac{\sqrt{-u^2+w^2}\tanh\left(1/2\,(K+\zeta)\sqrt{-u^2+w^2}\right)}{w} + 4\left(-\frac{u}{w} + \frac{\sqrt{-u^2+w^2}\tanh\left(\frac{1}{2}(K+\zeta)\sqrt{-u^2+w^2}\right)}{w}\right)^2\right]. \tag{56}$$

$$m_{15}(\zeta) = \ln\left[u^2 - \frac{-2\,uv + 2\,uw}{w(K+\zeta)} + \frac{v^2+w^2-2\,vw}{w^2(K+\zeta)^2}\right]. \tag{57}$$

$$m_{18}(\zeta) = \ln\left[1 + 2\,\frac{(v+w)e^{v(K+\zeta)}+1}{(v-w)e^{h(K+\zeta)}+1} + \frac{\left((v+w)e^{v(K+\zeta)}+1\right)^2}{\left((v-w)e^{h(C+\zeta)}+1\right)^2}\right]. \tag{58}$$

Figure 1 depicts the solutions of the generalized q-deformed Sinh-Gordon equation for set 1 graphically. After selecting the values of $\alpha = -0.5$, $K = 3$ that are suitable for the method.

6 Conclusion

In this paper, q-deformed Sinh-Gordon equation had been examined. The new extended generalized Kudryashov and improved $\tan(\frac{\Phi}{2})$-expansion approaches were successfully used to achieve additional and innovative exact traveling wave solutions. In the form of singular and rational type soliton solutions, solitary wave solutions and exact solutions of hyperbolic, exponential and rational functions had been created. Maple and Mathematica applications were used for computation and graphical representations. The novel results show that the recommended strategies were successful in identifying novel solutions to several nonlinear partial differential equations that arise in a variety of domains of applied nonlinear sciences.

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Declarations

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