Research Article

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A new computational investigation to the new exact solutions of (3 + 1)-dimensional WKdV equations *via* two novel procedures arising in shallow water magnetohydrodynamics

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Abstract: Various new exact solutions to (3 + 1)-dimensional Wazwaz–KdV equations are obtained in this work *via* two techniques: the modified Kudryashov procedure and modified simple equation method. The 3D plots, contour plots, and 2D plots of some obtained solutions are provided to describe the dynamic characteristics of the obtained solutions. Our employed techniques are very helpful in constructing new exact solutions to several nonlinear models encountered in ocean scientific phenomena arising in stratified flows, shallow water, plasma physics, and internal waves.

Keywords: exact solutions, (3 + 1)-dimensional modified Wazwaz–KdV equations, modified Kudryashov procedure, modified simple equation method

1 Introduction

In many physical phenomena, nonlinear partial differential equations (NPDEs) are apparent in modeling these phenomena [1]. To understand the dynamic behaviour of these models, several research studies have been dedicated to study the exact solutions of NLPDEqs using a variety of procedures such as the enhanced Kudryashov's (KdV) technique [2], general projective Riccati equations technique [2], sine-Gordon expansion technique [3,4], sinh-Gordon expansion technique [5,6], Hirota bilinear approach [7], Riccati–Bernoulli sub ordinary differential equation (ODE) technique [8], modified simple equation (MSE) technique [9], KdV and exponential techniques [10–12], and improved F-expansion technique [13]. In addition, some studies have formulated some of NPDEs in the sense of fractional calculus such as the fractional-order Kaup–Boussinesq and generalized Hirota Satsuma-coupled KdV systems [14], (3 + 1)-dimensional conformable Wazwaz–Benjamin–Bona–Mahony equation [15], and nonlinear fractional Schrödinger equation [16] (see also ref. [17]).

For the fractional version of NPDEs and other types of differential equations, a newly proposed definition of generalized fractional derivative, named Abu-Shady–Kaabar fractional derivative, [18], can be utilized further in studying these equations due to the simplicity and efficiency of obtained analytical solutions using this new definition.

This article is organized as follows: Basic preliminaries about our adapted algorithms are reviewed in Section 2. The utilized procedures, particularly the modified KdV and MSE procedures, are discussed in Section 3. The illustrations of some obtained solutions are represented graphically in Section 4. A conclusion is drawn in Section 5.

2 Adopted algorithms

The needed tools are presented here to help in a NPDE's reduction to an ODE. We suppose that NPDE is expressed as follows:

$$P(u, u_t, u_x, u_y, u_z, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0, \tag{1}$$

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where P is a polynomial of u and its partial derivatives.

We will take a transformation as follows:

$$\xi = kx + ry + sz - wt. \tag{2}$$

Here, k, r, s, and w are constants. By substituting Eq. (2) to Eq. (1), we obtain an ODE as follows, which will be integrated with respect to ξ possible times [19].

$$Q(u, u', u'', u''', \ldots) = 0. \tag{3}$$

In Sections 2.1 and 2.2, we describe the modified KdV and the modified simple eqaution (MSE) procedures, respectively.

2.1 The procedure of modified Kudryashov

The exact solutions of Eq. (3) are assumed as follows follows:

$$u(\xi) = \sum_{n=0}^{m} \alpha_i(\phi(\xi))^n. \tag{4}$$

Here, $\alpha_n(n = 0, 1, \ldots, m)$ are constants to be obtained later, and $\alpha_m \neq 0$. The balancing term is represented by m. $\phi(\xi)$ is written as follows:

$$\phi(\xi) = \frac{1}{1 + \delta a^{\xi}}, \tag{5}$$

and Eq. (5) satisfies:

$$\phi'(\xi) = \phi(\xi)(\phi(\xi) - 1)\ln a. \tag{6}$$

Nonlinear algebraic equations' system is obtained for $\alpha_n(n = 0, \ldots, m)$, a, k, r, s, and w by substituting Eq. (4) into Eq. (3) associated with Eq. (6) and then setting the collection of all the coefficients of $\phi^n(\xi)$ to be 0. By solving the obtained system *via* MAPLE, a variety of exact solutions [20–22] is found.

2.2 The method of MSE

We present the MSE method's main steps along with its fundamental ideas [27]. Through the transformation Eq. (2), Eq. (1) can be changed into Eq. (3). This procedure benefits from choosing the solution of Eq. (3) as follows:

$$u(\xi) = \sum_{n=0}^{m} a_n\left[\frac{\phi'(\xi)}{\phi(\xi)}\right]^n, \tag{7}$$

where a_n, $(n = 0, 1, 2, 3, \ldots, m)$ are arbitrary constants to be found later $\ni a_m \neq 0$, and $\phi(\xi)$ is an unknown function to be found later $\ni \phi'(\xi) \neq 0$. We may calculate the positive integer m occur in Eq. (7) *via* the homogeneous balance principle. Substituting Eq. (7) into Eq. (3), a polynomial in $\phi(\xi)$ can be obtained, and then by setting all the coefficients of $\phi^j(\xi) \quad (j = \ldots, -2, -1, 0)$ to 0 obtains nonlinear algebraic equation's system for a_n, $(n = 0, 1, 2, 3, \ldots, m)$ and $\phi(\xi)$. A variety of exact solutions is constructed for the desired equation *via* solving the obtained system.

Remark 1. The obtained solution *via* the tanh-function method, $\left(\frac{G'}{G}\right)$-expansion method, and exp-function method is expressed in the terms of some predefined functions, but in the MSE method, ϕ is not predefined or not a solution of any predefined equation. Therefore, novel solutions are obtained *via* this technique.

3 The modified version of (3 + 1)-dimensional KdV equations

The modified (3 + 1)-dimensional KdV equations' exact solutions are presented in this section. These equations are expressed as follows [23,24]:

$$u_t + 6u^2u_x + u_{xyz} = 0. \tag{8}$$

$$u_t + 6u^2u_y + u_{xyz} = 0. \tag{9}$$

$$u_t + 6u^2u_z + u_{xyz} = 0. \tag{10}$$

The above equations are essential in mathematical physics topics. The first equation is given by Hereman [25], while the second and third equations are given by Wazwaz [26].

3.1 Application of the modified Kudryashov procedure

We will employ the modified KdV procedure to the adopted equations.

3.1.1 First equation's exact solutions

Let the wave variable: $\xi = kx + ry + sz - wt$ be applied to Eq. (8). Then, by integrating the obtained ODE, we obtain:

$$-wu + 2ku^3 + krsu'' = 0. \tag{11}$$

Here, according to the homogeneous balance principle, the balancing number is 1. So, the ODE's solution is written as follows:

$$u(\xi) = a_0 + a_1\phi(\xi). \quad (12)$$

Eq. (12) is substituted *via* Eq. (6)'s help into Eq. (11). Then, by collecting all terms with the same power of $\phi(\xi)$, we obtain:

$$\begin{aligned}&(\phi(\xi))^3 : 2\ln(a)^2 krsa_1 + 2ka_1^3,\\&(\phi(\xi))^2 : -3\ln(a)^2 krsa_1 + 6ka_0a_1^2,\\&(\phi(\xi))^1 : \ln(a)^2 krsa_1 + 6ka_0^2a_1 - a_1w,\\&(\phi(\xi))^0 : 2ka_0^3 - wa_0.\end{aligned} \quad (13)$$

The exact solutions are obtained by solving the aforementioned system as follows:

$$a_0 = \pm\frac{\sqrt{-rs}\ln(a)}{2},\ a_1 = \pm\frac{rs\ln(a)}{\sqrt{-rs}},\ w = -\frac{ksr\ln(a)^2}{2}. \quad (14)$$

Then, Eq. (8)'s exact solutions are expressed as follows:

$$u_{1,2}(x,y,z,t) = \pm\frac{\sqrt{-rs}\ln(a)}{2} \pm \frac{rs\ln(a)}{\sqrt{-rs}\left(1+\delta a^{\left(kx+ry+sz+\frac{ksr\ln(a)^2}{2}t\right)}\right)}. \quad (15)$$

3.1.2 The second equation's exact solutions

Let the wave variable: $\xi = kx + ry + sz - wt$ be applied to Eq. (9). Then, by integrating the obtained ODE, we obtain:

$$-wu + 2ru^3 + krsu'' = 0. \quad (16)$$

Here, the balancing number is 1. So, the ODE's solution is same as Eq. (12). Eq. (12) is substituted *via* Eq. (6)'s help into Eq. (16). Then, by collecting all terms with the same power of $\phi(\xi)$, we obtain:

$$\begin{aligned}&(\phi(\xi))^3 : 2\ln(a)^2 krsa_1 + 2ra_1^3,\\&(\phi(\xi))^2 : -3\ln(a)^2 krsa_1 + 6ra_0a_1^2,\\&(\phi(\xi))^1 : \ln(a)^2 krsa_1 + 6ra_0^2a_1 - a_1w,\\&(\phi(\xi))^0 : 2ra_0^3 - wa_0.\end{aligned}$$

If we solve the aforementioned system, we obtain following values of the constant:

$$a_0 = \pm\frac{\sqrt{-ks}\ln(a)}{2},\ a_1 = \pm\frac{ks\ln(a)}{\sqrt{-ks}},\ w = -\frac{ksr\ln(a)^2}{2}. \quad (17)$$

Then, Eq. (9)'s exact solutions are expressed as follows:

$$u_{1,2}(x,y,z,t) = \pm\frac{\sqrt{-ks}\ln(a)}{2} \pm \frac{ks\ln(a)}{\sqrt{-ks}\left(1+\delta a^{\left(kx+ry+sz+\frac{ksr\ln(a)^2}{2}t\right)}\right)}. \quad (18)$$

3.1.3 The third equation's exact solutions

Let the wave variable: $\xi = kx + ry + sz - wt$ be applied to Eq. (10), and we obtain:

$$-wu + 2su^3 + krsu'' = 0. \quad (19)$$

Here, the balancing number is 1. So, the ODE's solution is same as Eq. (12). Eq. (12) is substituted *via* Eq. (6)'s help into Eq. (19). Then, by collecting all terms with the same power of $\phi(\xi)$, we obtain:

$$\begin{aligned}&(\phi(\xi))^3 : 2\ln(a)^2 krsa_1 + 2sa_1^3,\\&(\phi(\xi))^2 : -3\ln(a)^2 krsa_1 + 6sa_0a_1^2,\\&(\phi(\xi))^1 : \ln(a)^2 krsa_1 + 6sa_0^2a_1 - a_1w,\\&(\phi(\xi))^0 : 2sa_0^3 - wa_0.\end{aligned}$$

The values of constants are obtained by solving the aforementioned system as follows:

$$a_0 = \pm\frac{\sqrt{-ks}\ln(a)}{2},\ a_1 = \pm\frac{ks\ln(a)}{\sqrt{-ks}},\ w = -\frac{ksr\ln(a)^2}{2}.$$

Thus, the third equation's exact solutions are expressed as follows:

$$u_{1,2}(x,y,z,t) = \pm\frac{\sqrt{-kr}\ln(a)}{2} \pm \frac{kr\ln(a)}{\sqrt{-kr}\left(1+\delta a^{\left(kx+ry+sz+\frac{ksr\ln(a)^2}{2}t\right)}\right)}. \quad (20)$$

3.2 Application of the modified simple equation procedure

We will employ the MSE procedure to the adopted equations.

3.2.1 The first equation's exact solutions

From the employed technique, Eq. (11)'s exact solution is assumed as follows:

$$u(\xi) = a_0 + a_1\left(\frac{\phi'(\xi)}{\phi(\xi)}\right). \quad (21)$$

Eq. (21) is substituted into Eq. (11), and all terms with the same power of $\phi(\xi)$ are collected. Then, we obtain:

$$(\phi(\xi))^{-3} : 2ka_1^3 + 2krsa_1 = 0, \quad (22)$$

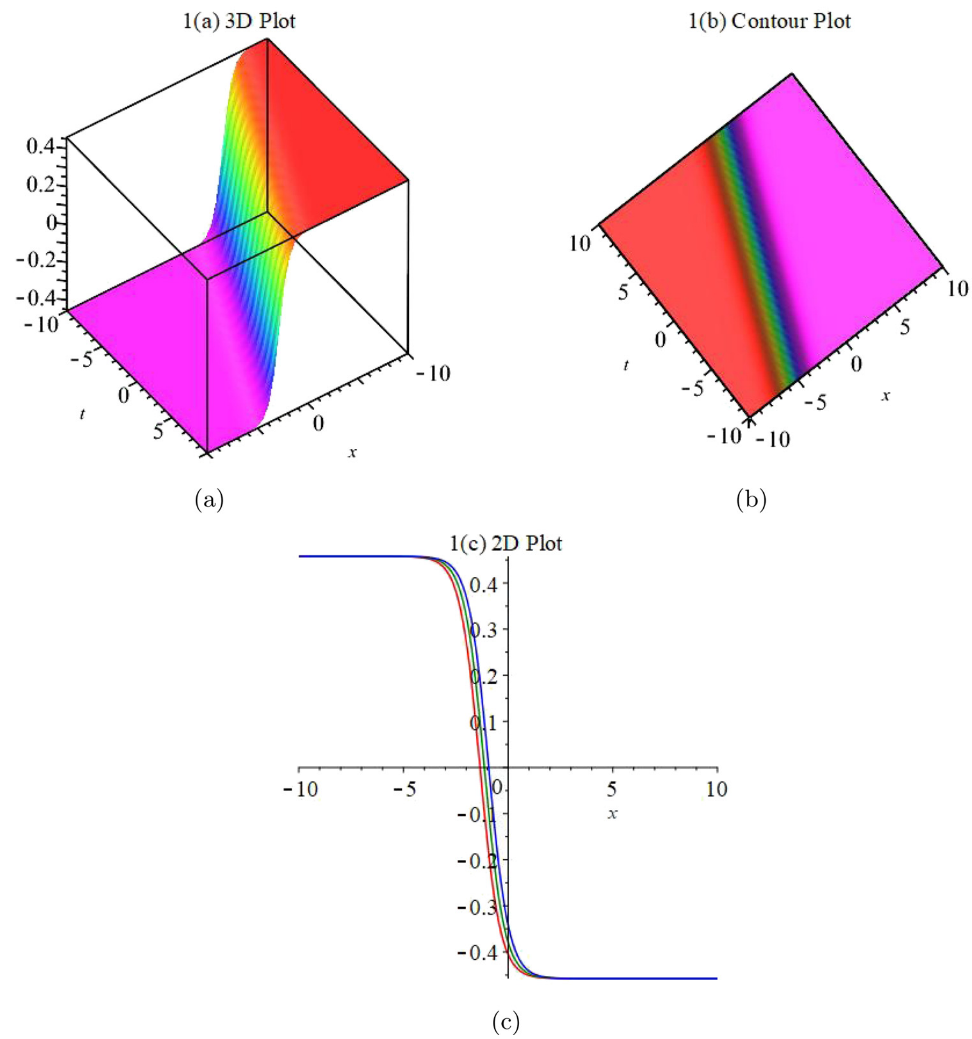

Figure 1: Graphical representations of (a) and (b) when $y = 1$, $z = 1$, $k = 2.1$, $r = -0.5$, $s = 1.7$, $a = 2.7$, $\delta = 5$; and (c) when $y = 1$, $z = 1$, $k = 2.1$, $r = -0.5$, $s = 1.7$, $a = 2.7$, $\delta = 5$. (a) 3D representation, (b) contour representation, and (c) 2D representation.

$$(\phi(\xi))^{-2} : 6ka_0a_1^2\phi'(\xi) - 3krsa_1\phi''(\xi) = 0, \tag{23}$$

$$(\phi(\xi))^{-1} : 6ka_1a_0^2\phi'(\xi) + krsa_1\phi'''(\xi) - wa_1\phi'(\xi) = 0, \tag{24}$$

$$(\phi(\xi))^{0} : 2ka_0^3 - wa_0 = 0. \tag{25}$$

By solving Eqs. (25) and (22), we obtain the following values of the constants:

$$a_0 = a_0,\ a_1 = \pm\sqrt{-rs},\ w = 2a_0^2k. \tag{26}$$

If we substitute Eq. (26) in Eqs. (23)–(24), we obtain:

$$\phi(\xi) = C_1 + C_2e^{\pm\frac{2a_0\sqrt{-rs}\xi}{rs}}, \tag{27}$$

and, Eq. (8)'s exact solutions are expressed as follows:

$$u_{3,4}(x, y, z, t) = a_0 - \frac{2C_2a_0e^{\pm\frac{2a_0\sqrt{-rs}\xi}{rs}}}{C_1 + C_2e^{\pm\frac{2a_0\sqrt{-rs}\xi}{rs}}}. \tag{28}$$

3.2.2 The second equation's exact solutions

From the employed technique, Eq. (16)'s exact solution is assumed as follows:

$$u(\xi) = a_0 + a_1\left(\frac{\phi'(\xi)}{\phi(\xi)}\right). \tag{29}$$

Eq. (29) is substituted into Eq. (16), and all terms with the same power of $\phi(\xi)$ are collected. Then, we obtain:

$$(\phi(\xi))^{-3} : 2ra_1^3 + 2krsa_1 = 0, \tag{30}$$

$$(\phi(\xi))^{-2} : 6ra_0a_1^2\phi'(\xi) - 3krsa_1\phi''(\xi) = 0, \tag{31}$$

$$(\phi(\xi))^{-1} : 6ra_1a_0^2\phi'(\xi) + krsa_1\phi'''(\xi) - wa_1\phi'(\xi) = 0, \tag{32}$$

$$(\phi(\xi))^{0} : 2ra_0^3 - wa_0 = 0. \tag{33}$$

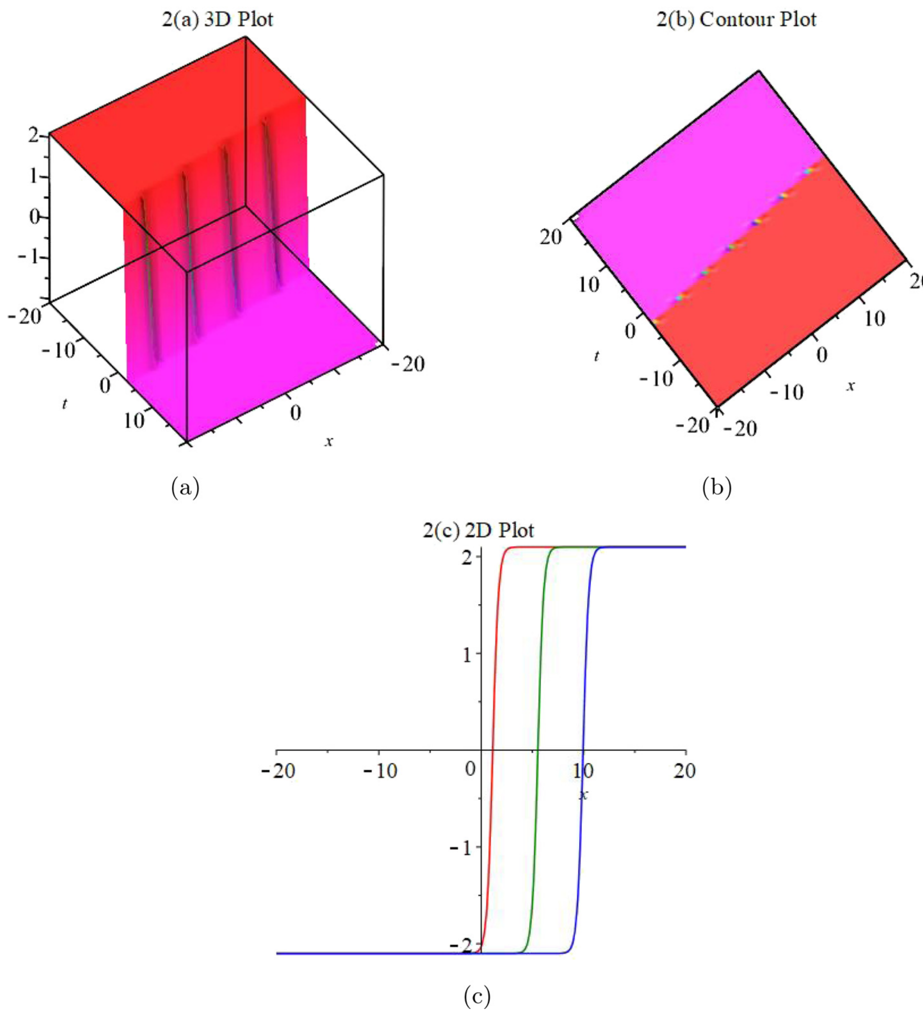

Figure 2: Graphical representations of (a) and (b) when $y = 1$, $z = 1$, $k = 2.1$, $r = -3.5$, $s = 1.7$, $C_1 = 0.3$, $C_2 = 0.8$, $a_0 = 2.1$; and (c) when $y = 1$, $z = 1$, $k = 2.1$, $r = -3.5$, $s = 1.7$, $C_1 = 0.3$, $C_2 = 0.8$, $a_0 = 2.1$. (a) 3D representation, (b) contour representation, and (c) 2D representation.

By solving Eqs. (30) and (33), we obtain the following values of the constants:

$$a_0 = a_0,\ a_1 = \pm\sqrt{-ks},\ w = 2a_0^2 r. \tag{34}$$

If we substitute Eq. (34) into Eqs. (31) and (32), we obtain:

$$\phi(\xi) = C_1 + C_2 e^{\pm\frac{2a_0\sqrt{-ks}\,\xi}{ks}}, \tag{35}$$

and, Eq. (9)'s exact solutions are as follows:

$$u_{3,4}(x, y, z, t) = a_0 - \frac{2C_2 a_0 e^{\pm\frac{2a_0\sqrt{-ks}\,\xi}{rs}}}{C_1 + C_2 e^{\pm\frac{2a_0\sqrt{-ks}\,\xi}{rs}}}. \tag{36}$$

3.2.3 The third equation's exact solutions

From the employed technique, Eq. (19)'s exact solution is assumed as follows:

$$u(\xi) = a_0 + a_1\left(\frac{\phi'(\xi)}{\phi(\xi)}\right). \tag{37}$$

Eq. (37) is substituted into Eq. (19), and all terms with the same power of $\phi(\xi)$ are collected. Then, we obtain:

$$(\phi(\xi))^{-3} : 2sa_1^3 + 2krsa_1 = 0, \tag{38}$$

$$(\phi(\xi))^{-2} : 6sa_0a_1^2\phi'(\xi) - 3krsa_1\phi''(\xi) = 0, \tag{39}$$

$$(\phi(\xi))^{-1} : 6sa_1a_0^2\phi'(\xi) + krsa_1\phi'''(\xi) - wa_1\phi'(\xi) = 0, \tag{40}$$

$$(\phi(\xi))^{0} : 2sa_0^3 - wa_0 = 0. \tag{41}$$

Solving Eqs. (38) and (41), we obtain the following values of the constants:

$$a_0 = a_0,\ a_1 = \pm\sqrt{-kr},\ w = 2a_0^2 s. \tag{42}$$

If we substitute Eq. (42) in Eqs. (39) and (40), we obtain:

$$\phi(\xi) = C_1 + C_2 e^{\pm\frac{2a_0\sqrt{-kr}\xi}{kr}}, \tag{43}$$

and Eq. (10)'s exact solutions are as follows:

$$u_{3,4}(x,y,z,t) = a_0 - \frac{2C_2 a_0 e^{\pm\frac{2a_0\sqrt{-kr}\xi}{kr}}}{C_1 + C_2 e^{\pm\frac{2a_0\sqrt{-kr}\xi}{kr}}}. \tag{44}$$

Remark 2. Here, we did not consider the case of $a_0 = 0$ as it leads to zero solutions.

4 Graphical representation of the obtained solutions

The figures of some obtained solutions are given in this section, which are obtained by the discussed methods. We give graphical illustrations by 3D plots, contour plots, and 2D plots.

First, we have given graphs for solution (8) in Figure 1. Figure 1(a) and (b) show 3D and contour plots, respectively. We have plotted them when $y = 1$, $z = 1$, $k = 2.1$, $r = -0.5$, $s = 1.7$, $a = 2.7$, and $\delta = 5$. Figure 1(c) shows the 2D plot which is plotted when $y = 1$, $z = 1$, $k = 2.1$, $r = -0.5$, $s = 1.7$, $a = 2.7$, and $\delta = 5$. Red line is plotted when $t = 0$, green line is plotted when $t = 0.5$, and blue line is plotted when $t = 1$.

Then, we have given graphs for solution (28) in Figure 2. Figure 2(a) and (b) show 3D and contour plots, respectively. We have plotted them when $y = 1$, $z = 1$, $k = 2.1$, $r = -3.5$, $s = 1.7$, $C_1 = 0.3$, $C_2 = 0.8$, $a_0 = 2.1$. Figure 2(c) shows the 2D plot, which is plotted when $y = 1$, $z = 1$, $k = 2.1$, $r = -3.5$, $s = 1.7$, $C_1 = 0.3$, $C_2 = 0.8$, and $a_0 = 2.1$. Red line is plotted when $t = 0$, green line is plotted when $t = 0.5$, and blue line is plotted when $t = 1$.

5 Conclusion

Exploring the exact solutions of NPDEs is essential in studying various modeling scenarios. In our work, we have obtained the (3 + 1)-dimensional Wazwaz-KdV (WKdV) equations' new exact solutions *via* the modified Kudryashov procedure and the MSE method. The graphical representations involving 3D plots, contour plots and 2D plots have been provided for some obtained solutions to understand the nonlinear model's behavior. These investigated techniques can provide exact solutions to many nonlinear models in physics and engineering. Future research works can be based on our results by extending this work to study the fractional version of (3 + 1)-dimensional WKdV equations *via* some new fractional definitions such as Abu-Shady–Kaabar fractional derivative [18].

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