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New exact solutions of some nonlinear evolution equations of pseudoparabolic type

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Abstract This paper aims to conduct an analytical study into some nonlinear models of pseudoparabolic type, including the Oskolkov, Oskolkov–Benjamin–Bona–Mahony–Burgers, and Benjamin–Bona–Mahony–Peregrine–Burgers equations. A number of new exact solutions for these pseudoparabolic type equations have been derived based on the modified Kudryashov method that its calculations are performed in a symbolic computation system known as *Maple*.

Keywords Pseudoparabolic type equations · Oskolkov equation · Oskolkov–Benjamin–Bona–Mahony–Burgers equation · Benjamin–Bona–Mahony–Peregrine–Burgers equation · Modified Kudryashov method · New exact solutions

1 Introduction

In general, nonlinear partial differential equations (NPDEs) consisting of high order terms play a vital role in describing complicated problems that are arisen in various scientific topics, such as thermodynamics, optical fibers, nonlinear networks, soil consolidation, fluid flow in rock discontinuities, and wave propagation. Thus, solving NPDEs is of crucial importance for researchers such that the resulting solutions can provide a great insight into

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the physical behavior of the problems concerned. During the recent years, exact solutions to NPDEs have been found by developing a broad range of analytical approaches, such as modified simple equation method (Kaplan and Bekir 2016; Younis 2014; Kaplan et al. 2015; Jawad et al. 2010), trial equation method (Ekici et al. 2016; Pandir et al. 2013; Bulut et al. 2013; Odabasi and Misirli 2015), first integral method (Hosseini and Gholamin 2015; Hosseini et al. 2012, 2014; Younis 2013), (G'/G)-expansion method (Younis and Zafar 2014; Biswas and Mirzazadeh 2014; Younis and Rizvi 2015; Khan et al. 2015), exp-function method (Bekir et al. 2015; Guner et al. 2015, 2016), and Kudryashov method (Kudryashov 2012; Ryabov 2010; Hosseini and Ayati 2016; Ayati et al. 2016; Eslami and Mirzazadeh 2014). Kudryashov method is regarded as one of the most powerful algebraic techniques for solving high order nonlinear differential equations. Recently, a variant of Kudryashov method in which the Euler's number e is replaced by an arbitrary constant $a \neq 1$ as the base of the exponential function has attracted a great deal of attention among researchers. For example, Hosseini et al. (2017) utilized the modified Kudryashov technique to extract new exact solutions of the conformable time-fractional Klein–Gordon equations with quadratic and cubic nonlinearities. Hosseini et al. (2017) also used the modified Kudryashov technique to generate new explicit exact solutions of the conformable time-fractional Cahn–Allen and Cahn–Hilliard equations. Zayed and Alurrfi (2015) exerted the modified Kudryashov technique to produce new exact solutions for some classes of seventh-order PDEs, used in mathematical physics, including Sawada–Kotera–Ito, Kaup–Kupershmidt, and Lax equations. More references are found in (Bulut et al. 2013, 2016 Saha 2016; Ege and Misirli 2014; Demiray et al. 2014; Mirzazadeh et al. 2016; Taghizadeh et al. 2017; Manafian 2017; Tariq and Akram 2017; Sahoo and Saha 2016; Çenesiz et al. 2016). In this study, a modified form of Kudryashov method has been suggested to seek new exact solutions of some pseudoparabolic type equations, namely the Oskolkov, Oskolkov–Benjamin–Bona–Mahony–Burgers, and Benjamin–Bona–Mahony–Peregrine–Burgers equations. Apart from this study, Gözükizil and Akçağil (2013) investigated the pseudoparabolic type equations using the tanh-coth method and Akcagil et al. (2016) studied the pseudoparabolic type equations via the (G'/G)-expansion method.

2 Describing the modified Kudryashov method

A general nonlinear partial differential equation in two independent variables x and t can be illustrated as

$$F\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial t^2}, \frac{\partial^2 u}{\partial x^2}, \ldots\right) = 0, \tag{1}$$

where F is a given polynomial of $u(x, t)$ along with its various partial derivatives. Under the transformation

$$u(x,t) = f(\xi), \quad \xi = l_2 x - l_1 t,$$

where two parameters l_1 and l_2 are nonzero constants to be evaluated later, Eq. (1) can be written as a nonlinear ordinary differential equation in the form

$$G(f, f', f'', \ldots) = 0. \tag{2}$$

Here $'$ shows the derivatives with respect to ξ. Assume that the solution of Eq. (2) is presented as follows

$$f(\xi) = \sum_{i=0}^{m} a_i Q^i(\xi), \tag{3}$$

where a_i, $i = 0, 1, \ldots, m$ are constants to be calculated, $a_m \neq 0$, and $Q = Q(\xi)$ is the solution of

$$\frac{dQ}{d\xi} = Q(\xi)(Q(\xi) - 1)\ln a.$$

It can be easily verified that the function $Q(\xi)$ have the form

$$Q(\xi) = \frac{1}{1 + da^{\xi}},$$

where d is an arbitrary constant and $a \neq 1$. To obtain the value of m, we have to balance the highest order derivative and nonlinear terms in Eq. (2). By substituting Eq. (3) into Eq. (2) and collecting the terms, we obtain

$$T(Q(\xi)) = 0. \tag{4}$$

By comparing the coefficients of like powers of $Q(\xi)$ in the resulting Eq. (4), a system of algebraic equations will be gained. Solving the system of equations through the use of symbolic computation system yields the values of a_i, l_1, and l_2. Subsequently, replacing these values into Eq. (3) produces a variety of new solutions for Eq. (1).

3 Application

In this section, the utility of the modified Kudryashov technique is assessed by exerting it to some nonlinear pseudoparabolic type equations, including the Oskolkov, Oskolkov–Benjamin–Bona–Mahony–Burgers, and Benjamin–Bona–Mahony–Peregrine–Burgers equations.

3.1 The Oskolkov equation

The Oskolkov equation is presented as (Gözükizil and Akçağil 2013; Akcagil et al. 2016)

$$u_t - k_1 u_{xxt} - k_2 u_{xx} + u u_x = 0, \tag{5}$$

which demonstrates the dynamic behavior of a Kelvin–Voigt model of viscoelastic fluids. By using the transformation introduced in Sect. 2, Eq. (5) can be reduced as

$$-l_1 f' + k_1 l_1 l_2^2 f''' - k_2 l_2^2 f'' + l_2 f f' = 0.$$

Integrating above equation once with respect to ξ yields

$$-l_1 f + k_1 l_1 l_2^2 f'' - k_2 l_2^2 f' + \frac{1}{2} l_2 f^2 = 0. \tag{6}$$

By balancing the derivative term f'' and the second-power term f^2, we find $m = 2$. This gives a finite series in the form

$$f(\xi) = a_0 + a_1 Q(\xi) + a_2 Q^2(\xi). \tag{7}$$

Substituting Eq. (7) along with its first and second derivatives into Eq. (6) and equating the coefficients of like powers of $Q(\xi)$ in the resulting equation, results in

$$\begin{aligned}
& - l_1 a_0 + \frac{1}{2} l_2 a_0^2 = 0, \\
& k_1 l_1 l_2^2 (\ln a)^2 a_1 + k_2 l_2^2 (\ln a) a_1 + l_2 a_0 a_1 - l_1 a_1 = 0, \\
& k_1 l_1 l_2^2 (\ln a)^2 a_1 + k_2 l_2^2 (\ln a) a_1 + l_2 a_0 a_1 - l_1 a_1 = 0, \\
& - l_1 a_2 - 3 k_1 l_1 l_2^2 (\ln a)^2 a_1 + 4 k_1 l_1 l_2^2 (\ln a)^2 a_2 - k_2 l_2^2 (\ln a) a_1 \\
& \quad + 2 k_2 l_2^2 (\ln a) a_2 + l_2 a_0 a_2 + \frac{1}{2} l_2 a_1^2 = 0, \\
& 2 k_1 l_1 l_2^2 (\ln a)^2 a_1 - 10 k_1 l_1 l_2^2 (\ln a)^2 a_2 - 2 k_2 l_2^2 (\ln a) a_2 + l_2 a_1 a_2 = 0, \\
& 6 k_1 l_1 l_2^2 (\ln a)^2 a_2 + \frac{1}{2} l_2 a_2^2 = 0.
\end{aligned}$$

The following different cases are determined by solving above system.

Case 1:

$$\begin{aligned}
& a_0 = \mp \frac{2\sqrt{6} k_2}{5\sqrt{k_1}}, \quad a_1 = 0, \quad a_2 = \pm \frac{2\sqrt{6} k_2}{5\sqrt{k_1}}, \\
& l_1 = -\frac{k_2}{5 k_1 \ln a}, \quad l_2 = \pm \frac{\sqrt{6}}{6\sqrt{k_1} \ln a}.
\end{aligned}$$

Now, the following new exact solutions to the Oskolkov equation are extracted

$$u_{1,2}(x,t) = \mp \frac{2\sqrt{6} k_2}{5\sqrt{k_1}} \pm \frac{2\sqrt{6} k_2}{5\sqrt{k_1} \left(1 + d a^{\pm \frac{\sqrt{6}}{6\sqrt{k_1} \ln a} x + \frac{k_2}{5 k_1 \ln a} t} \right)^2}.$$

Case 2:

$$\begin{aligned}
& a_0 = 0, \quad a_1 = 0, \quad a_2 = \pm \frac{12 k_2}{5\sqrt{-6 k_1}}, \\
& l_1 = -\frac{k_2}{5 k_1 \ln a}, \quad l_2 = \pm \frac{1}{\sqrt{-6 k_1} \ln a}.
\end{aligned}$$

Now, the following new exact solutions to the Oskolkov equation are generated

$$u_{3,4}(x,t) = \pm \frac{12 k_2}{5\sqrt{-6 k_1} \left(1 + d a^{\pm \frac{1}{\sqrt{-6 k_1} \ln a} x + \frac{k_2}{5 k_1 \ln a} t} \right)^2}.$$

Case 3:

$$a_0 = \mp\frac{12k_2}{5\sqrt{-6k_1}}, \quad a_1 = \pm\frac{24k_2}{5\sqrt{-6k_1}}, \quad a_2 = \mp\frac{12k_2}{5\sqrt{-6k_1}},$$
$$l_1 = \frac{k_2}{5k_1 \ln a}, \quad l_2 = \pm\frac{1}{\sqrt{-6k_1}\ln a}.$$

Now, the following new exact solutions to the Oskolkov equation are produced

$$u_{5,6}(x,t) = \mp\frac{12k_2}{5\sqrt{-6k_1}} \pm \frac{24k_2}{5\sqrt{-6k_1}\left(1 + da^{\pm\frac{1}{\sqrt{-6k_1}\ln a}x - \frac{k_2}{5k_1 \ln a}t}\right)} \mp \frac{12k_2}{5\sqrt{-6k_1}\left(1 + da^{\pm\frac{1}{\sqrt{-6k_1}\ln a}x - \frac{k_2}{5k_1 \ln a}t}\right)^2}.$$

Case 4:

$$a_0 = 0, \quad a_1 = \pm\frac{4\sqrt{6}k_2}{5\sqrt{k_1}}, \quad a_2 = \mp\frac{2\sqrt{6}k_2}{5\sqrt{k_1}},$$
$$l_1 = \frac{k_2}{5k_1 \ln a}, \quad l_2 = \pm\frac{\sqrt{6}}{6\sqrt{k_1}\ln a}.$$

Now, the following new exact solutions to the Oskolkov equation are constructed

$$u_{7,8}(x,t) = \pm\frac{4\sqrt{6}k_2}{5\sqrt{k_1}\left(1 + da^{\pm\frac{\sqrt{6}}{6\sqrt{k_1}\ln a}x - \frac{k_2}{5k_1 \ln a}t}\right)} \mp \frac{2\sqrt{6}k_2}{5\sqrt{k_1}\left(1 + da^{\pm\frac{\sqrt{6}}{6\sqrt{k_1}\ln a}x - \frac{k_2}{5k_1 \ln a}t}\right)^2}.$$

3.2 The Oskolkov–Benjamin–Bona–Mahony–Burgers equation

The Oskolkov–Benjamin–Bona–Mahony–Burgers equation is defined as (Gözükizil and Akçağil 2013; Akcagil et al. 2016)

$$u_t - u_{xxt} - k_1 u_{xx} + k_2 u_x + k_3 u u_x = 0. \tag{8}$$

This equation presents nonlinear surface waves that circumfuse along the horizontal axis Ox. Under the transformation

$$u(x,t) = f(\xi), \quad \xi = l_2 x - l_1 t,$$

the Oskolkov–Benjamin–Bona–Mahony–Burgers (OBBMB) Eq. (8) is reduced to a nonlinear ODE as below

$$(k_2 l_2 - l_1)f' - k_1 l_2^2 f'' + l_1 l_2^2 f''' + k_3 l_2 f f' = 0. \tag{9}$$

Integrating Eq. (9) once with respect to ξ gives

$$(k_2 l_2 - l_1)f - k_1 l_2^2 f' + l_1 l_2^2 f'' + \frac{1}{2}k_3 l_2 f^2 = 0. \tag{10}$$

In a similar manner as before, we find $m = 2$. This provides a solution of the form

$$f(\xi) = a_0 + a_1 Q(\xi) + a_2 Q^2(\xi). \tag{11}$$

Setting Eq. (11) along with its first and second derivatives into Eq. (10) and comparing the coefficients of same powers of $Q(\xi)$ in the resulting equation, yields

$$\begin{aligned}
&k_2 l_2 a_0 - l_1 a_0 + \frac{1}{2} k_3 l_2 a_0^2 = 0,\\
&l_1 l_2^2 (\ln a)^2 a_1 + k_1 l_2^2 (\ln a) a_1 + k_3 l_2 a_0 a_1 + k_2 l_2 a_1 - l_1 a_1 = 0,\\
&k_2 l_2 a_2 - l_1 a_2 - 3 l_1 l_2^2 (\ln a)^2 a_1 + 4 l_1 l_2^2 (\ln a)^2 a_2\\
&\quad - k_1 l_2^2 (\ln a) a_1 + 2 k_1 l_2^2 (\ln a) a_2 + k_3 l_2 a_0 a_2 + \frac{1}{2} k_3 l_2 a_1^2 = 0,\\
&2 l_1 l_2^2 (\ln a)^2 a_1 - 10 l_1 l_2^2 (\ln a)^2 a_2 - 2 k_1 l_2^2 (\ln a) a_2 + k_3 l_2 a_1 a_2 = 0,\\
&6 l_1 l_2^2 (\ln a)^2 a_2 + \frac{1}{2} k_3 l_2 a_2^2 = 0.
\end{aligned}$$

By solving above nonlinear system through the use of symbolic computation system, the following cases are determined.

Case 1:

$$a_0 = -\frac{2\left(5k_2\sqrt{24k_1^2 + 25k_2^2} \pm \left(12k_1^2 + 25k_2^2\right)\right)}{5k_3\left(\sqrt{24k_1^2 + 25k_2^2} \pm 5k_2\right)}, \quad a_1 = 0, \quad a_2 = \frac{5k_2 \pm \sqrt{24k_1^2 + 25k_2^2}}{5k_3},$$

$$l_1 = -\frac{k_1}{5\ln a}, \quad l_2 = \frac{5k_2 \pm \sqrt{24k_1^2 + 25k_2^2}}{12k_1 \ln a}.$$

Now, the following new exact solutions to the OBBMB equation are generated

$$u_{1,2}(x,t) = -\frac{2\left(5k_2\sqrt{24k_1^2 + 25k_2^2} \pm \left(12k_1^2 + 25k_2^2\right)\right)}{5k_3\left(\sqrt{24k_1^2 + 25k_2^2} \pm 5k_2\right)} + \frac{5k_2 \pm \sqrt{24k_1^2 + 25k_2^2}}{5k_3\left(1 + d a^{\frac{5k_2 \pm \sqrt{24k_1^2 + 25k_2^2}}{12k_1 \ln a} x + \frac{k_1}{5\ln a} t}\right)^2}.$$

Case 2:

$$a_0 = 0, \quad a_1 = 0, \quad a_2 = \frac{-5k_2 \pm \sqrt{-24k_1^2 + 25k_2^2}}{5k_3},$$

$$l_1 = -\frac{k_1}{5\ln a}, \quad l_2 = \frac{-5k_2 \pm \sqrt{-24k_1^2 + 25k_2^2}}{12k_1 \ln a}.$$

Now, the following new exact solutions to the OBBMB equation are extracted

$$u_{3,4}(x,t) = \frac{-5k_2 \pm \sqrt{-24k_1^2 + 25k_2^2}}{5k_3\left(1 + d a^{\frac{-5k_2 \pm \sqrt{-24k_1^2 + 25k_2^2}}{12k_1 \ln a} x + \frac{k_1}{5\ln a} t}\right)^2}.$$

Case 3:

$$a_0 = 0, \quad a_1 = \frac{2\left(-5k_2 \pm \sqrt{24k_1^2 + 25k_2^2}\right)}{5k_3}, \quad a_2 = \frac{5k_2 \mp \sqrt{24k_1^2 + 25k_2^2}}{5k_3},$$
$$l_1 = \frac{k_1}{5\ln a}, \quad l_2 = \frac{-5k_2 \pm \sqrt{24k_1^2 + 25k_2^2}}{12k_1 \ln a}.$$

Now, the following new exact solutions to the OBBMB equation are produced

$$u_{5,6}(x,t) = \frac{2\left(-5k_2 \pm \sqrt{24k_1^2 + 25k_2^2}\right)}{5k_3\left(1 + da^{\frac{-5k_2 \pm \sqrt{24k_1^2+25k_2^2}}{12k_1 \ln a}x - \frac{k_1}{5\ln a}t}\right)} + \frac{5k_2 \mp \sqrt{24k_1^2 + 25k_2^2}}{5k_3\left(1 + da^{\frac{-5k_2 \pm \sqrt{24k_1^2+25k_2^2}}{12k_1 \ln a}x - \frac{k_1}{5\ln a}t}\right)^2}.$$

3.3 The Benjamin–Bona–Mahony–Peregrine–Burgers equation

The Benjamin–Bona–Mahony–Peregrine–Burgers equation is expressed as (Gözükizil and Akçağil 2013; Akcagil et al. 2016)

$$u_t - u_{xxt} - k_1 u_{xx} + k_2 u_x + k_3 u u_x + k_4 u_{xxx} = 0. \tag{12}$$

By introducing the transformation

$$u(x,t) = f(\xi), \xi = l_2 x - l_1 t,$$

the Benjamin–Bona–Mahony–Peregrine–Burgers (BBMPB) (12) turns into a nonlinear ODE of the form

$$(k_2 l_2 - l_1)f' - k_1 l_2^2 f'' + \left(l_1 l_2^2 + k_4 l_2^3\right)f''' + k_3 l_2 f f' = 0. \tag{13}$$

Integrating above Eq. (13) once with respect to ξ results in

$$(k_2 l_2 - l_1)f - k_1 l_2^2 f' + \left(l_1 l_2^2 + k_4 l_2^3\right)f'' + \frac{1}{2}k_3 l_2 f^2 = 0. \tag{14}$$

As in the preceding equations, we obtain $m = 2$. Thus, Eq. (3) takes a form as below

$$f(\xi) = a_0 + a_1 Q(\xi) + a_2 Q^2(\xi). \tag{15}$$

Setting Eq. (15) along with its first and second derivatives into Eq. (14) and comparing the coefficients of like powers of $Q(\xi)$, leads to

$$k_2 l_2 a_0 + \frac{1}{2}k_3 l_2 a_0^2 - l_1 a_0 = 0, k_4 l_2^3 (\ln a)^2 a_1 + l_1 l_2^2 (\ln a)^2 a_1 + k_1 l_2^2 (\ln a) a_1 + k_3 l_2 a_0 a_1$$
$$+ k_2 l_2 a_1 - l_1 a_1 = 0, -l_1 a_2 + k_3 l_2 a_0 a_2 - 3k_4 l_2^3 (\ln a)^2 a_1 - 3l_1 l_2^2 (\ln a)^2 a_1 + 4k_4 l_2^3 (\ln a)^2 a_2$$
$$+ 4l_1 l_2^2 (\ln a)^2 a_2 - k_1 l_2^2 (\ln a) a_1 + 2k_1 l_2^2 (\ln a) a_2 + k_2 l_2 a_2 + \frac{1}{2}k_3 l_2 a_1^2 = 0, 2k_4 l_2^3 (\ln a)^2 a_1$$
$$- 10k_4 l_2^3 (\ln a)^2 a_2 + 2l_1 l_2^2 (\ln a)^2 a_1 - 10 l_1 l_2^2 (\ln a)^2 a_2 - 2k_1 l_2^2 (\ln a) a_2 + k_3 l_2 a_1 a_2 = 0,$$
$$6k_4 l_2^3 (\ln a)^2 a_2 + 6l_1 l_2^2 (\ln a)^2 a_2 + \frac{1}{2}k_3 l_2 a_2^2 = 0.$$

The following different cases are determined by solving above system.
Case 1:

$$a_0 = 0, \quad a_1 = -\frac{2(k_2 + k_4)}{k_3}, \quad a_2 = 0, \quad l_1 = \frac{k_4(k_2 + k_4)}{k_1 \ln a}, \quad l_2 = -\frac{k_2 + k_4}{k_1 \ln a}.$$

Now, the following new exact solution to the BBMPB equation is produced

$$u_1(x,t) = -\frac{2(k_2 + k_4)}{k_3\left(1 + da^{-\frac{k_2+k_4}{k_1 \ln a}x - \frac{k_4(k_2+k_4)}{k_1 \ln a}t}\right)}.$$

Case 2:

$$a_0 = -\frac{2(k_2 + k_4)}{k_3}, \quad a_1 = \frac{2(k_2 + k_4)}{k_3}, \quad a_2 = 0,$$
$$l_1 = -\frac{k_4(k_2 + k_4)}{k_1 \ln a}, \quad l_2 = \frac{k_2 + k_4}{k_1 \ln a}.$$

Now, the following new exact solution to the BBMPB equation is extracted

$$u_2(x,t) = -\frac{2(k_2 + k_4)}{k_3} + \frac{2(k_2 + k_4)}{k_3\left(1 + da^{\frac{k_2+k_4}{k_1 \ln a}x + \frac{k_4(k_2+k_4)}{k_1 \ln a}t}\right)}.$$

Case 3:

$$a_0 = 0, \quad a_1 = 0, \quad a_2 = \frac{-5k_2 - 5k_4 \pm \sqrt{-24k_1^2 + (5(k_2 + k_4))^2}}{5k_3},$$
$$l_1 = \frac{-12k_1^2 + 25k_2k_4 + 25k_4^2 \mp 5k_4\sqrt{-24k_1^2 + (5(k_2 + k_4))^2}}{60k_1 \ln a},$$
$$l_2 = \frac{-5k_2 - 5k_4 \pm \sqrt{-24k_1^2 + (5(k_2 + k_4))^2}}{12k_1 \ln a}.$$

Now, the following new exact solutions to the BBMPB equation are extracted

$$u_{3,4}(x,t) = \frac{-5k_2 - 5k_4 \pm \sqrt{-24k_1^2 + (5(k_2 + k_4))^2}}{5k_3\left(1 + da^{\frac{-5k_2-5k_4 \pm \sqrt{-24k_1^2+(5(k_2+k_4))^2}}{12k_1 \ln a}x - \frac{-12k_1^2+25k_2k_4+25k_4^2 \mp 5k_4\sqrt{-24k_1^2+(5(k_2+k_4))^2}}{60k_1 \ln a}t}\right)^2}.$$

4 Conclusion

This paper suggested a modified form of Kudrayshov method and also validated the effectiveness of this technique by exerting it to different nonlinear pseudoparabolic type equations, including the Oskolkov, Oskolkov–Benjamin–Bona–Mahony–Burgers,and

Benjamin–Bona–Mahony–Peregrine–Burgers equations. As a direct result of this analysis, a number of new exact solutions were produced. The correctness of each solution was then checked by putting them back into its corresponding equation, ensuring that this new technique is widely applicable to various differential equations of nonlinear type.

References

Akcagil, S., Aydemir, T., Gozukizil, O.F.: Exact travelling wave solutions of nonlinear pseudoparabolic equations by using the G'/G expansion method. New Trends Math. Sci. **4**, 51–66 (2016)

Ayati, Z., Hosseini, K., Mirzazadeh, M.: Application of Kudryashov and functional variable methods to the strain wave equation in microstructured solids. Nonlinear Eng. (2016). doi:10.1515/nleng-2016-0020

Bekir, A., Güner, Ö., Bhrawy, A.H., Biswas, A.: Solving nonlinear fractional differential equations using exp-function and G'/G expansion methods. Rom. J. Phys. **60**, 360–378 (2015)

Biswas, A., Mirzazadeh, M.: Dark optical solitons with power law nonlinearity using (G'/G)-expansion. Optik **125**, 4603–4608 (2014)

Bulut, H., Baskonus, H.M., Pandir, Y.: The modified trial equation method for fractional wave equation and time fractional generalized Burgers equation. Abstr. Appl. Anal. **2013**, 636802 (2013a)

Bulut, H., Pandir, Y., Baskonus, H.M.: Symmetrical hyperbolic Fibonacci function solutions of generalized Fisher equation with fractional order. AIP Conf. Proc. **1558**, 1914 (2013b)

Bulut, H., Sulaiman, T.A., Baskonus, H.M.: New solitary and optical wave structures to the Korteweg–de Vries equation with dual-power law nonlinearity. Opt. Quant. Electron. **48**, 564 (2016). doi:10.1007/s11082-016-0831-4

Çenesiz, Y., Baleanu, D., Kurt, A., Tasbozan, O.: New exact solutions of Burgers' type equations with conformable derivative. Waves Random Complex Media (2016). doi:10.1080/17455030.2016.1205237

Demiray, S.T., Pandir, Y., Bulut, H.: Generalized Kudryashov method for time-fractional differential equations. Abstr. Appl. Anal. **2014**, 901540 (2014)

Ege, S.M., Misirli, E.: The modified Kudryashov method for solving some fractional-order nonlinear equations. Adv. Differ. Equ. **2014**, 135 (2014). doi:10.1186/1687-1847-2014-135

Ekici, M., Mirzazadeh, M., Zhou, Q., Moshokoa, S.P., Biswas, A., Belic, M.: Solitons in optical metamaterials with fractional temporal evolution. Optik **127**, 10879–10897 (2016)

Eslami, M., Mirzazadeh, M.: Exact solutions for fifth-order KdV-type equations with time-dependent coefficients using the Kudryashov method. Eur. Phys. J. Plus **129**, 192 (2014). doi:10.1140/epjp/i2014-14192-1

Gözükizil, O.F., Akçağil, Ş.: The tanh-coth method for some nonlinear pseudoparabolic equations with exact solutions. Adv. Differ. Equ. **2013**, 143 (2013). doi:10.1186/1687-1847-2013-143

Guner, O., Bekir, A., Bilgil, H.: A note on exp-function method combined with complex transform method applied to fractional differential equations. Adv. Nonlinear Anal. **4**, 201–208 (2015)

Guner, O., Aksoy, E., Bekir, A., Cevikel, A.C.: Different methods for (3 + 1)-dimensional space-time fractional modified KdV–Zakharov–Kuznetsov equation. Comput. Math Appl. **71**, 1259–1269 (2016)

Hosseini, K., Ayati, Z.: Exact solutions of space-time fractional EW and modified EW equations using Kudryashov method. Nonlinear Sci. Lett. A **7**, 58–66 (2016)

Hosseini, K., Gholamin, P.: Feng's first integral method for analytic treatment of two higher dimensional nonlinear partial differential equations. Differ. Equ. Dyn. Syst. **23**, 317–325 (2015)

Hosseini, K., Ansari, R., Gholamin, P.: Exact solutions of some nonlinear systems of partial differential equations by using the first integral method. J. Math. Anal. Appl. **387**, 807–814 (2012)

Hosseini, K., Sadeghi, F., Ansari, R.: First integral method for solving nonlinear physical systems of partial differential equations. J. Nat. Sci. Sustain. Tech. **8**, 391–400 (2014)

Hosseini, K., Mayeli, P., Ansari, R.: Modified Kudryashov method for solving the conformable time-fractional Klein–Gordon equations with quadratic and cubic nonlinearities. Optik **130**, 737–742 (2017a)

Hosseini, K., Bekir, A., Ansari, R.: New exact solutions of the conformable time-fractional Cahn–Allen and Cahn–Hilliard equations using the modified Kudryashov method. Optik **132**, 203–209 (2017b)

Jawad, A.J.M., Petkovic, M.D., Biswas, A.: Modified simple equation method for nonlinear evolution equations. Appl. Math. Comput. **217**, 869–877 (2010)

Kaplan, M., Bekir, A.: The modified simple equation method for solving some fractional-order nonlinear equations. Pramana J. Phys. **87**, 15 (2016). doi:10.1007/s12043-016-1205-y

Kaplan, M., Bekir, A., Akbulut, A., Aksoy, E.: The modified simple equation method for nonlinear fractional differential equations. Rom. J. Phys. **60**, 1374–1383 (2015)

Khan, K., Ali Akbar, M., Rashidi, M.M., Zamanpour, I.: Exact traveling wave solutions of an autonomous system via the enhanced (G'/G)-expansion method. Waves Random Complex Media **25**, 644–655 (2015)

Kudryashov, N.A.: One method for finding exact solutions of nonlinear differential equations. Commun. Nonlinear Sci. Numer. Simul. **17**, 2248–2253 (2012)

Manafian, J.: Application of the ITEM for the system of equations for the ion sound and Langmuir waves. Opt. Quant. Electron. **49**, 17 (2017). doi:10.1007/s11082-016-0860-z

Mirzazadeh, M., Ekici, M., Zhou, Q., Sonmezoglu, A.: Analytical study of solitons to the generalized resonant dispersive nonlinear Schrödinger's equation with power law nonlinearity. Superlattices Microstruct. (2016). doi:10.1016/j.spmi.2016.12.003

Odabasi, M., Misirli, E.: On the solutions of the nonlinear fractional differential equations via the modified trial equation method. Math. Methods Appl. Sci. (2015). doi:10.1002/mma.3533

Pandir, Y., Gurefe, Y., Misirli, E.: The extended trial equation method for some time fractional differential equations. Discrete Dyn. Nat. Soc. **2013**, 491359 (2013)

Ryabov, P.N.: Exact solutions of the Kudryashov–Sinelshchikov equation. Appl. Math. Comput. **217**, 3585–3590 (2010)

Saha, R.S.: New analytical exact solutions of time fractional KdV–KZK equation by Kudryashov methods. Chin. Phys. B **25**, 040204 (2016)

Sahoo, S., Saha, S.: Ray, Solitary wave solutions for time fractional third order modified KdV equation using two reliable techniques (G'/G)-expansion method and improved (G'/G)-expansion method. Phys. A **448**, 265–282 (2016)

Taghizadeh, N., Zhou, Q., Ekici, M., Mirzazadeh, M.: Soliton solutions for Davydov solitons in α-helix proteins. Superlattices Microstruct. **102**, 323–341 (2017)

Tariq, H., Akram, G.: New approach for exact solutions of time fractional Cahn–Allen equation and time fractional Phi-4 equation. Phys. A (2017). doi:10.1016/j.physa.2016.12.081

Younis, M.: The first integral method for time-space fractional differential equations. J. Adv. Phys. **2**, 220–223 (2013)

Younis, M.: A new approach for the exact solutions of nonlinear equations of fractional order via modified simple equation method. Appl. Math. **5**, 1927–1932 (2014)

Younis, M., Rizvi, S.T.R.: Dispersive dark optical soliton in $(2 + 1)$-dimensions by (G'/G)-expansion with dual-power law nonlinearity. Optik **126**, 5812–5814 (2015)

Younis, M., Zafar, A.: Exact solution to nonlinear differential equations of fractional order via (G/G')-expansion method. Appl. Math. **5**, 1–6 (2014)

Zayed, E.M.E., Alurrfi, K.A.E.: The modified Kudryashov method for solving some seventh order nonlinear PDEs in mathematical physics. World J. Model. Simul. **11**, 308–319 (2015)