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ORIGINAL PAPER

A generalized Kudryashov method to some nonlinear evolution equations in mathematical physics

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Abstract Nonlinear evolution equations form the most fundamental theme in mathematical physics. The search for exact solutions of nonlinear equations has been of interest in recent years. In this paper, we obtain exact solutions of the nonlinear Jaulent–Miodek hierarchy and (2+1)-dimensional Calogero–Bogoyavlenskii–Schiff equation by using the generalized Kudryashov method. All calculations in this study have been made and checked back with the aid of the Maple packet program.

Keywords Exact solutions · Generalized Kudryashov method · Symbolic computation · Calogero–Bogoyavlenskii–Schiff equation · Jaulent–Miodek hierarchy

1 Introduction

In the past four decades, the research area of nonlinear evolution equations modeling various physical phenomena has played a significant role in a great many applications such as fluid mechanics, water waves, plasma physics, nonlinear optics, elastic media, solid-state physics and acoustic waves in crystals. A large amount of effort has been expended over the last ten years or so in attempting to find robust and stable analytical methods to solve these equations. Many powerful methods have been presented to investigate exact solutions of nonlinear equations, such as the homogeneous balance method [1], the inverse scattering method [2], the Backlund transformation method [3,4], Hirota bilinear method [5], Wronskian technique [6], Darboux transformation [7], Lie group analysis [8–10], Jacobi elliptic function method [11], the tanh method [12], extended tanh method [13,14], sine–cosine method [15,16], F-expansion method [17,18], variational iteration method [19], exp-function method [20,21], the (G'/G)-expansion method [22,23], the $(G'/G, 1/G)$-expansion method [24–26], the auxiliary equation method [27–29], the trial method [30], the modified simple equation method [31,32], the ansatz method [33–36], multiple exp-function algorithm [37] and so on [38].

The study of multidimensional integrable systems is one of the main topics in theory of the integrable systems. Several integrable models have been recently developed in the context of (2+1)-dimensional equations. Our objective in this work is to obtain new traveling wave solutions of two (2+1)-dimensional nonlinear partial differential equation (NPDE) by using the generalized Kudryashov method . The proposed method is a direct and powerful method for solving NPDEs.

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The present paper organized as follows: In Sect. 2, we describe the generalized Kudryashov method to find exact solutions of NPDEs. In Sect. 3, we illustrate this method in detail with the Jaulent–Miodek hierarchy and Calogero–Bogoyavlenskii–Schiff (CBS) equation. Also we discuss the graphical representation of some obtained solutions. Finally, in Sect. 4, some conclusions are given.

2 The generalized Kudryashov method

In this section, we present the generalized Kudryashov method for finding exact solutions of NPDEs [39,40].

We consider a general NPDE in the form:

$$P(u, u_x, u_y, u_t, u_{xx}, u_{yy}, ...) = 0, \tag{1}$$

where P is a polynomial of $u(x, y, z, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved.

The main steps of the generalized Kudryashov method are as follows:

Step 1: To look for the traveling wave solution of equation, we take traveling wave transformation:

$$\xi = x + y + z + \lambda t \tag{2}$$

By using Eq. (2), Eq. (1) reduces to a nonlinear ordinary differential equation (ODE).

$$H(u, u', u'', ...,) = 0, \tag{3}$$

where prime denotes the derivation with respect to ξ.

Step 2: Suppose that the solution of Eq. (3) can be represented by in the following rational form:

$$u(\xi) = \frac{\sum_{i=0}^{N} a_i Q^i(\xi)}{\sum_{j=0}^{M} b_j Q^j(\xi)} \tag{4}$$

where $a_i (i = 0, 1, ..., n), b_j (j = 0, 1, ..., m)$ are constants to be determined such that $a_N \neq 0, b_M \neq 0$ and $Q = Q(\xi)$ satisfies the ODE

$$\frac{dQ}{d\xi} = Q^2(\xi) - Q(\xi) \tag{5}$$

It is obvious that the solution of Eq. (5) is

$$Q(\xi) = \frac{1}{1 + Ae^{\xi}}$$

where A is integration constant.

Step 3: We determine the positive integers N and M in Eq. (4) by using the homogeneous balance method between the highest order derivatives and the highest power nonlinear terms in Eq. (3).

Step 4: Substituting Eq. (4) into Eq. (3) along with Eq. (5), we obtain a polynomial of $R(Q)$ of Q. Then equating all the coefficients of $R(Q)$ to zero, we acquire a system of algebraic equations. Solving this system with the aid of Maple, we can find the values of $a_i (i = 0, 1, ..., n), b_j (j = 0, 1, ..., m)$. Finally, if we substitute these values and Eq. (5) into Eq. (4), then we find the exact solutions of the reduced Eq. (3).

3 Applications

3.1 The Jaulent–Miodek hierarchy

The equation generated by the Jaulent–Miodek hierarchy is in the following form

$$w_t + \frac{1}{4}\left(w_{xx} - 2w^3\right)_x + \frac{3}{4}\left(\frac{1}{4}\partial_x^{-1} w_{yy} + w_x \partial_x^{-1} w_y\right) = 0, \tag{6}$$

Here ∂_x^{-1} denotes an inverse operator of $\partial_x = \partial/\partial_x$, which can be defined as $\partial_x^{-1} f(x) = \int_{-\infty}^{x} f(t)dt$ under the decaying condition at infinity with the condition $\partial_x^{-1}\partial_x = \partial_x\partial_x^{-1} = 1$ [41].

To remove the integral term, assume

$$w(x, y, t) = u_x(x, y, t) \tag{7}$$

therefore, Eq. (6) becomes the following form:

$$u_{xt} + \frac{1}{4}u_{xxxx} - \frac{3}{2}u_x^2 u_{xx} + \frac{3}{16}u_{yy} + \frac{3}{4}u_{xx}u_y = 0 \tag{8}$$

Let us apply the generalized Kudryashov method to find the exact solutions of the Jaulent–Miodek hierarchy. Assume $u(x, y, t) = u(\xi)$, $\xi = x + y + \lambda t$, where λ is the wave speed. Equation (8) reduces to:

$$\lambda u'' + \frac{1}{4}u'''' - \frac{3}{2}(u')^2u'' + \frac{3}{16}u'' + \frac{3}{4}u''u' = 0 \tag{9}$$

Then Eq. (9) can be rearranged as follows:

$$\lambda u'' + \frac{1}{4}u'''' - \frac{1}{2}((u')^3)' + \frac{3}{16}u'' + \frac{3}{8}((u')^2)' = 0. \tag{10}$$

Integrating Eq. (10) with respect to ξ and setting the integration constant to zero, we get:

$$\lambda u' + \frac{1}{4}u''' - \frac{1}{2}(u')^3 + \frac{3}{16}u' + \frac{3}{8}(u')^2 = 0. \tag{11}$$

By substituting $u' = v$ into Eq. (11), we obtain

$$\left(\lambda + \frac{3}{16}\right) v + \frac{1}{4}v'' - \frac{1}{2}v^3 + \frac{3}{8}v^2 = 0. \tag{12}$$

Considering the homogeneous balance between the highest order derivative term v'' with the highest order nonlinear term v^3 gives

$$N = M + 1$$

If we choose $M = 1$, then $N = 2$. Therefore, the exact solution of the proposed equation takes the form

$$v(\xi) = \frac{a_0 + a_1 Q + a_2 Q^2}{b_0 + b_1 Q} \tag{13}$$

where $Q = Q(\xi)$ satisfies Eq. (5) and a_0, a_1, a_2, b_0, b_1 are parameters to be determined. Substituting Eq. (13) into Eq. (12) and using Eq. (5) and then equating all coefficients of the functions Q^k to zero, we obtain

$$Q^6 : \frac{1}{2}a_2b_1^2 - \frac{1}{2}a_2^3 = 0,$$

$$Q^5 : \frac{3}{8}a_2^2b_1 - \frac{3}{8}a_1a_2^2 + \frac{3}{2}a_2b_0b_1 - \frac{3}{4}a_2b_1^2 = 0,$$

$$\begin{aligned} Q^4 : {}& \frac{3}{2}a_2b_0^2 - \frac{9}{4}a_2b_0b_1 + \frac{3}{8}a_2^2b_0 - \frac{3}{2}a_0a_2^2 - \frac{3}{2}a_1^2a_2 \\ & + \frac{7}{16}a_2b_1 + \lambda a_2b_1^2 + \frac{3}{4}a_1a_2b_1 = 0, \end{aligned}$$

$$\begin{aligned} Q^3 : {}& \frac{9}{8}a_2b_0b_1 + \frac{1}{2}a_1b_0^2 + 2\lambda a_2b_0b_1 - \frac{1}{4}a_0b_1^2 \\ & + \frac{3}{8}a_1^2b_1 + \frac{3}{16}a_1b_1^2 + \lambda a_1b_1^2 \\ & - \frac{5}{2}a_2b_0^2 - \frac{1}{2}a_1^3 + \frac{3}{4}a_1a_2b_0 + \frac{1}{4}a_1b_0b_1 \\ & - 3a_0a_1a_2 - \frac{1}{2}a_0b_0b_1 + \frac{3}{4}a_0a_2b_1 = 0, \end{aligned}$$

$$\begin{aligned} Q^2 : {}& \frac{3}{8}a_1^2b_0 + \frac{3}{4}a_0a_2b_0 + \frac{7}{16}a_0b_1^2 \\ & + 2\lambda a_1b_0b_1 + \lambda a_0b_1^2 + \frac{1}{8}a_1b_0b_1 \\ & + \frac{3}{4}a_0a_1b_1 - \frac{3}{4}a_1b_0^2 - \frac{3}{2}a_0a_1^2 + \frac{19}{16}a_2b_0^2 \\ & - \frac{3}{2}a_0^2a_2 + \lambda a_2b_0^2 + \frac{3}{4}a_0b_0b_1 = 0, \end{aligned}$$

$$\begin{aligned} Q^1 : {}& \frac{3}{4}a_0a_1b_0 + 2\lambda a_0b_0b_1 + \frac{7}{16}a_1b_0^2 + \lambda a_1b_0^2 \\ & - \frac{3}{2}a_0^2a_1 + \frac{3}{8}a_0^2b_1 + \frac{1}{8}a_0b_0b_1 = 0, \end{aligned}$$

$$Q^0 : \frac{3}{8}a_0^2b_0 + \frac{3}{16}a_0b_0^2 + \lambda a_0b_0^2 - \frac{1}{2}a_0^3 = 0.$$

Solving the above set of algebraic equations by Maple, we get different cases of the solutions. We discuss them below:

Case 1:

$$a_0 = 0, a_1 = -4b_0, a_2 = 4b_0, b_0 = b_0, b_1 = -4b_0, \lambda = -\frac{7}{16}.$$

By substituting these values into Eq. (13), we get

$$v(x, y, t) = \frac{-4\left(\frac{1}{1+C_1e^{x+y-\frac{7}{16}t}}\right) + 4\left(\frac{1}{1+C_1e^{x+y-\frac{7}{16}t}}\right)^2}{1 - 4\left(\frac{1}{1+C_1e^{x+y-\frac{7}{16}t}}\right)}.$$

Here C_1 is an integration constant. Since $u' = v$, we obtain the exact solution of Eq. (8) as follows (Fig. 1):

$$u(x, y, t) = \ln\left|\frac{1 + C_1\left(\cosh(x + y - \frac{7}{16}t) + \sinh(x + y - \frac{7}{16}t)\right)}{C_1\left(\cosh(x + y - \frac{7}{16}t) + \sinh(x + y - \frac{7}{16}t)\right) - 3}\right| + C,$$

where C is the integration constant. As $w = u_x$, the exact solution of the Jaulent–Miodek hierarchy is

$$\begin{aligned} w(x, y, t) = {}& -\frac{C1\left(\cosh(x + y - \frac{7}{16}t) + \sinh(x + y - \frac{7}{16}t)\right)}{-3 + C1\left(\cosh(x + y - \frac{7}{16}t) + \sinh(x + y - \frac{7}{16}t)\right)} \\ & + \frac{C1\left(\cosh(x + y - \frac{7}{16}t) + \sinh(x + y - \frac{7}{16}t)\right)}{1 + C1\left(\cosh(x + y - \frac{7}{16}t) + \sinh(x + y - \frac{7}{16}t)\right)}. \end{aligned}$$

Case 2:

$$a_0 = 0, a_1 = \frac{4}{3}b_0, a_2 = -\frac{4}{3}b_0, b_0 = b_0,$$

$$b_1 = -\frac{4}{3}b_0, \lambda = -\frac{7}{16}.$$

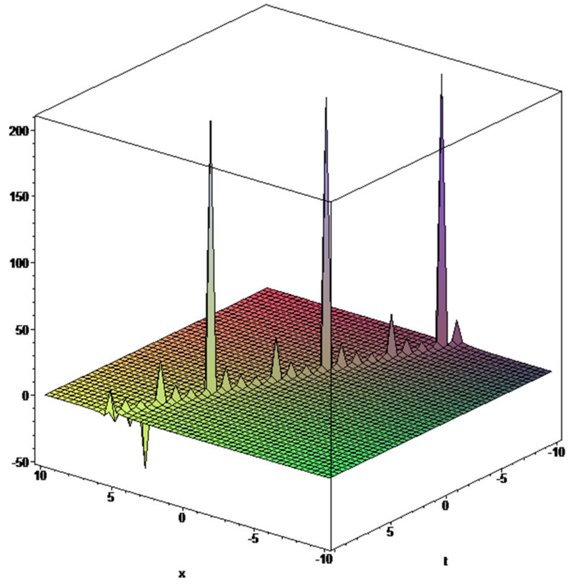

Fig. 1 Graph of the $w(x, y, t)$ corresponding to the values for $C_1 = 1$, $y = 0$ within the interval $-10 \leq t \leq 10$

Substituting these values into Eq. (13), we get

$$v(x, y, t) = \frac{4C_1 e^{x+y-\frac{7}{16}t}}{\left(-1+3C_1 e^{x+y-\frac{7}{16}t}\right)\left(1+C_1 e^{x+y-\frac{7}{16}t}\right)}$$

Here C_1 is an integration constant. Since $u' = v$, we obtain the exact solution of Eq. (8) as follows (Fig. 2):

$$u(x, y, t) = \ln\left|\frac{-1+3C_1\left(\cosh(x+y-\frac{7}{16}t)+\sinh(x+y-\frac{7}{16}t)\right)}{1+C_1\left(\cosh(x+y-\frac{7}{16}t)+\sinh(x+y-\frac{7}{16}t)\right)}\right| + 2C,$$

where C is the integration constant. As $w = u_x$, the exact solution of the Jaulent–Miodek hierarchy is

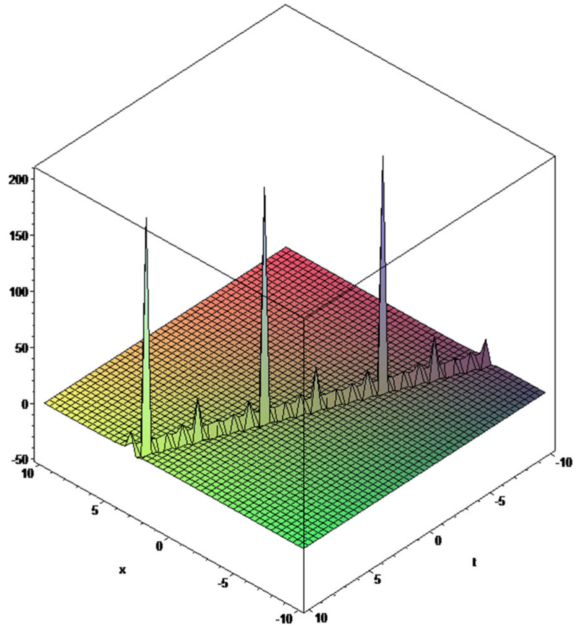

Fig. 2 Graph of the $w(x, y, t)$ corresponding to the values for $C_1 = 1$, $y = 0$ within the interval $-10 \leq t \leq 10$

$$w(x, y, t) = -\frac{C1\left(\cosh(x+y-\frac{7}{16}t)+\sinh(x+y-\frac{7}{16}t)\right)}{1+C1\left(\cosh(x+y-\frac{7}{16}t)+\sinh(x+y-\frac{7}{16}t)\right)} + \frac{3C1\left(\cosh(x+y-\frac{7}{16}t)+\sinh(x+y-\frac{7}{16}t)\right)}{-1+3C1\left(\cosh(x+y-\frac{7}{16}t)+\sinh(x+y-\frac{7}{16}t)\right)}.$$

Case 3:

$$a_0 = \frac{b_1(23+3\sqrt{73})}{128}, a_1 = \frac{-b_1(13+\sqrt{73})}{16},$$

$$b_0 = \frac{-b_1(9+\sqrt{73})}{16}, \lambda = \frac{3\sqrt{73}-25}{256}$$

Substituting these values into Eq. (13), we get

$$v(x,y,t)=\frac{\frac{23+3\sqrt{73}}{128}-\frac{13+\sqrt{73}}{16+16C_1\left(\cosh(x+y+\frac{3\sqrt{73}-25}{256}t)+\sinh(x+y+\frac{3\sqrt{73}-25}{256}t)\right)}}{\frac{1}{1+C_1\left(\cosh(x+y+\frac{3\sqrt{73}-25}{256}t)+\sinh(x+y+\frac{3\sqrt{73}-25}{256}t)\right)}-\frac{9+\sqrt{73}}{16}}$$
$$+\frac{\frac{1}{\left(1+C_1(\cosh(x+y+\frac{3\sqrt{73}-25}{256}t)+\sinh(x+y+\frac{3\sqrt{73}-25}{256}t))\right)^2}}{\frac{1}{1+C_1\left(\cosh(x+y+\frac{3\sqrt{73}-25}{256}t)+\sinh(x+y+\frac{3\sqrt{73}-25}{256}t)\right)}-\frac{9+\sqrt{73}}{16}}$$

Here C_1 is an integration constant. Since $u'=v$, we obtain the exact solution of Eq. (8) as follows (Fig. 3):

$$u(x,y,t)=\frac{-272\ln\left(-17+C_1A(x,y,t)+2\sqrt{73}\right)+32\sqrt{73}\ln\left(-17+C_1A(x,y,t)+2\sqrt{73}\right)}{16(-17+2\sqrt{73})}$$
$$+\frac{-197\ln A(x,y,t)+23\sqrt{73}\ln A(x,y,t)+272ln(1+C_1A(x,y,t))-32\sqrt{73}\ln(1+A(x,y,t)C1)}{16(-17+2\sqrt{73})}$$

where $A(x,y,t)=\cosh(x+y+\frac{3\sqrt{73}-25}{256}t)+\sinh(x+y+\frac{3\sqrt{73}-25}{256}t)$ and C is the integration constant. As $w=u_x$, the exact solution of the Jaulent–Miodek hierarchy is

$$w(x,y,t)=-\frac{C_1e^{x+y+\frac{3\sqrt{73}-25}{256}t}}{1+e^{x+y+\frac{3\sqrt{73}-25}{256}t}}+\frac{47+5\sqrt{73}}{56-8\sqrt{73}}+\frac{C_1e^{x+y+\frac{3\sqrt{73}-25}{256}t}}{-17+C_1e^{x+y+\frac{3\sqrt{73}-25}{256}t}+2\sqrt{73}}$$

Note that the obtained solutions are different from the given ones in [42].

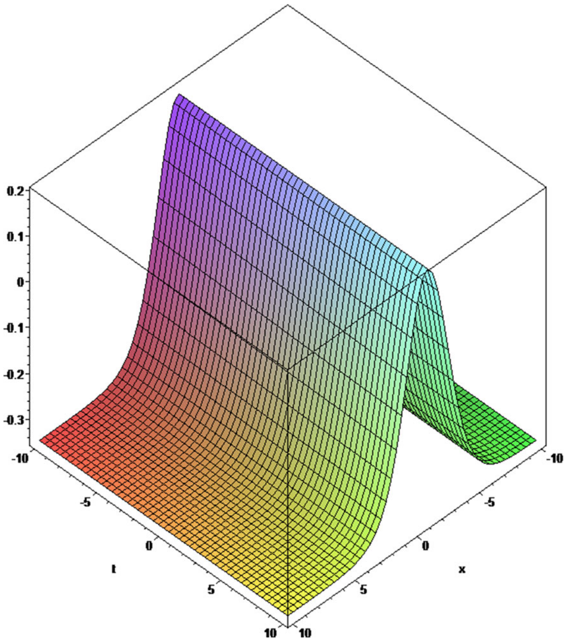

Fig. 3 Graph of the $w(x,y,t)$ corresponding to the values for $C_1=1$, $y=0$ within the interval $-10\le t\le 10$

3.2 Calogero–Bogoyavlenskii–Schiff equation

The (2 + 1)-dimensional CBS equation is in the form [43]:

$$u_{xxxz}+2u_zu_{xx}+4u_xu_{xz}+u_{xt}=0. \tag{14}$$

Let us solve Eq. (14) by using the generalized Kudryashov method. To achieve it, we first take a traveling wave transformation:

$$u(x,z,t)=u(\xi),\xi=x+z+\lambda t. \tag{15}$$

Therefore, Eq. (14) reduces to the following ODE:

$$u''''+3((u')^2)'+\lambda u''=0. \tag{16}$$

Here $'$ denotes the derivation with respect to ξ. Integrating Eq. (14) once with respect to ξ and setting the constant of the integration, we get

$$u''' + 3(u')^2 + \lambda u' = 0. \tag{17}$$

Setting $u' = v$, we obtain

$$v'' + 3v^2 + \lambda v = 0. \tag{18}$$

Balancing the highest order derivative term v'' with the nonlinear term v^2 gives

$N = M + 2.$

If we choose $M = 1$, then $N = 3$ and the exact solution takes the form

$$v(\xi) = \frac{a_0 + a_1 Q + a_2 Q^2 + a_3 Q^3}{b_0 + b_1 Q} \tag{19}$$

where $Q = Q(\xi)$ satisfies Eq. (5) and $a_0, a_1, a_2, a_3, b_0, b_1$ are parameters to be determined. Substituting Eq. (19) into Eq. (18) along with Eq. (5) and then equating all coefficients of the functions Q^k to zero, we obtain:

$$\begin{aligned}
Q^7 &: 3a_3^2 b_1 + 6a_3 b_1^2 = 0,\\
Q^6 &: 3a_3^2 b_0 + 16a_3 b_0 b_1 + 6a_2 a_3 b_1 - 10a_3 b_1^2\\
&\quad + 2a_2 b_1^2 = 0,\\
Q^5 &: 12a_3 b_0^2 + 6a_2 b_0 b_1 + 6a_2 a_3 b_0 + 4a_3 b_1^2 + \lambda a_3 b_1^2\\
&\quad - 3a_2 b_1^2 - 27a_3 b_0 b_1 + 6a_1 a_3 b_1 + 3a_2^2 b_1 = 0,\\
Q^4 &: 3a_2^2 b_0 + 11a_3 b_0 b_1 + 6a_2 b_0^2 + 2\lambda a_3 b_0 b_1\\
&\quad + 6a_0 a_3 b_1 + 6a_1 a_3 b_0\\
&\quad - 21a_3 b_0^2 - 9a_2 b_0 b_1 + 6a_1 a_2 b_1 + \lambda a_2 b_1^2\\
&\quad + a_2 b_1^2 = 0,\\
Q^3 &: 2\lambda a_2 b_0 b_1 + 6a_1 a_2 b_0 + 2a_1 b_0^2 - 10a_2 b_0^2\\
&\quad + 3a_2 b_0 b_1 + a_1 b_0 b_1 + \lambda a_3 b_0^2\\
&\quad + 9a_3 b_0^2 - b_1^2 a_0 - 2b_1 a_0 b_0 + 6a_0 a_3 b_0\\
&\quad + 3a_1^2 b_1 + 6a_0 a_2 b_1 + \lambda a_1 b_1^2 = 0,\\
Q^2 &: -a_1 b_0 b_1 - 3a_1 b_0^2 + 6a_0 a_1 b_1 + 6a_0 a_2 b_0\\
&\quad + \lambda a_2 b_0^2 + 2\lambda a_1 b_0 b_1\\
&\quad + b_1^2 a_0 + 3b_1 a_0 b_0 + \lambda b_1^2 a_0 + 3a_1^2 b_0\\
&\quad + 4a_2 b_0^2 = 0,\\
Q^1 &: 2\lambda b_1 a_0 b_0 + 6a_0 a_1 b_0 + \lambda a_1 b_0^2 + 3a_0^2 b_1 + a_1 b_0^2\\
&\quad - b_1 a_0 b_0 = 0,\\
Q^0 &: \lambda a_0 b_0^2 + 3a_0^2 b_0 = 0.
\end{aligned}$$

Solving above set of algebraic equations by Maple, we obtain (Fig. 4):

Case 1:

$a_0 = 0, a_1 = 2b_0, a_2 = -2b_0 + 2b_1, a_3 = -2b_1, \lambda = -1.$

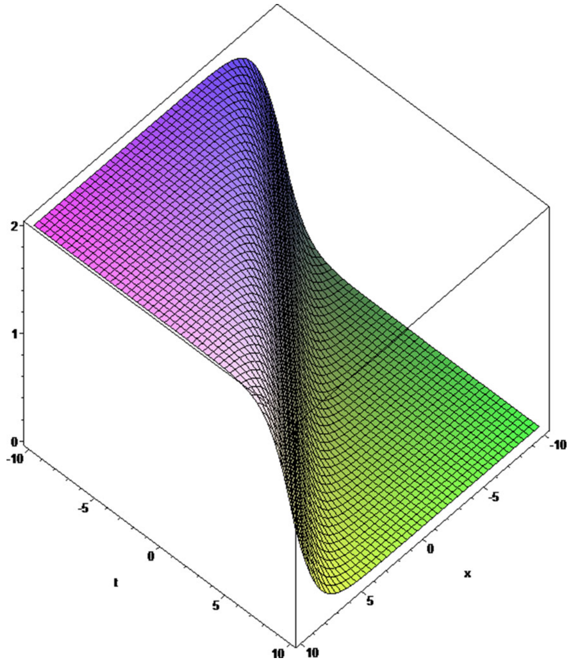

Fig. 4 Graph of the $u(x, z, t)$ corresponding to the values for $A = 1$, $B = 2$, $z = 0$ within the interval $-10 \leq t \leq 10$

Substituting these values into Eq. (19), we find

$$v(\xi) = \frac{2f(\xi)}{(1 + f(\xi))^2}.$$

Since $u' = v$, we get the exact solution of Eq. (14):

$$u(\xi) = B - \frac{2}{1 + f(\xi)},$$

where $f(\xi) = A(\cosh \xi + \sinh \xi)$, $\xi = x + z - t$ and B is integration constant.

Case 2:

$a_0 = -\frac{1}{3} b_0, a_1 = 2b_0 - \frac{1}{3} b_1, a_2 = -2b_0 + 2b_1,$
$a_3 = -2b_1, \lambda = 1.$

Substituting these values into Eq. (19), we find

$$v(\xi) = \frac{-Af(2\xi) + 4f(\xi) - 1}{3(1 + f(\xi))^2}.$$

Since $u' = v$, we get the exact solution of Eq. (14):

$$u(\xi) = \frac{1}{3} \ln\left(\frac{A}{f(\xi)}\right) + B - 2,$$

where $f(\xi) = A(\cosh \xi + \sinh \xi)$, $\xi = x + z + t$, and B is integration constant (Fig. 5).

Note that the obtained solutions are different from the ones given in [44,45].

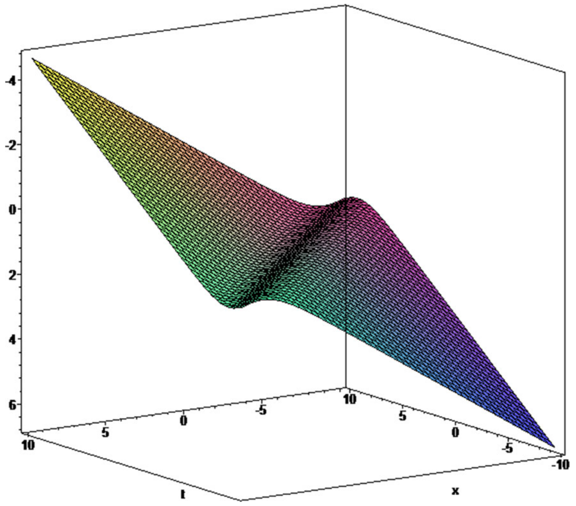

Fig. 5 Graph of the $u(x, z, t)$ corresponding to the values for $A = 1$, $B = 2$, $z = 0$ within the interval $-10 \leq t \leq 10$

4 Conclusions

In summary, we have presented the generalized Kudryashov method to construct exact solutions of nonlinear evolution equations. We have obtained the exact solutions of Jaulent–Miodek hierarchy and the CBS equation. The obtained solutions may be appropriate to understand the mechanism of the intricate nonlinear physical phenomena in wave collaboration. The generalized Kudryashov method is very effective and powerful to obtain exact solutions of nonlinear evolution equations and give more general solutions. In fact, this method is readily applicable to a large variety of the nonlinear evolution equations which frequently appear in mathematical physics and nonlinear sciences. By substituting back, it has been checked that the obtained solutions satisfy both the Jaulent–Miodek hierarchy and the CBS equation. Also the graphical representation of the obtained solutions are given.

References

1. Wang, M.L.: Solitary wave solutions for variant Boussinesq equations. Phys. Lett. A **199**, 169–172 (1995)
2. Ablowitz, M.J., Segur, H.: Solitons and Inverse Scattering Transformation. SIAM, Philadelphia (1981)
3. Zedan, H.A.: Exact solutions for the generalized KdV equation by using Backlund transformations. J. Frankl. Inst. **348**, 1751–1768 (2011)
4. Lü, X., Tian, B., Zhang, H.-Q., Xu, T., Li, H.: Generalized (2 + 1)-dimensional Gardner model: bilinear equations. Bäcklund transformation, Lax representation and interaction mechanisms. Nonlinear Dyn. **67**, 2279–2290 (2012)
5. Wazwaz, A.M.: Multiple-soliton solutions for the Boussinesq equation. Appl. Math. Comput. **192**(2), 479–486 (2007)
6. Ma, W.X., Abdeljabbar, A., Asaad, M.G.: Wronskian and Grammian solutions to a (3+1)-dimensional generalized KP equation. Appl. Math. Comput. **217**, 10016–10023 (2011)
7. Lü, X., Lin, F.: Soliton excitations and shape-changing collisions in alpha helical proteins with interspine coupling at higher order. Commun. Nonlinear Sci. Numer. Simul. **32**, 241–261 (2016)
8. Biswas, A., Khalique, C.M.: Stationary solutions for nonlinear dispersive Schrödinger's equation. Nonlinear Dyn. **63**, 623–626 (2011)
9. Biswas, A., Kara, A.H., Bokhari, A.H., Zaman, F.D.: Solitons and conservation laws of Klein–Gordon equation with power law and log law nonlinearities. Nonlinear Dyn. **73**, 2191–2196 (2013)
10. Adem, A.R., Lü, X.: Travelling wave solutions of a two-dimensional generalized Sawada–Kotera equation. Nonlinear Dyn. **84**, 915–922 (2016)
11. Adem, A.R., Muatjetjeja, B.: Conservation laws and exact solutions for a 2D Zakharov–Kuznetsov equation. Appl. Math. Lett. **48**, 109–117 (2015)
12. Wazwaz, A.M.: The tanh method for travelling wave solutions of nonlinear equations. Appl. Math. Comput. **154**(3), 713–723 (2004)
13. Fan, E.: Extented tanh-function method and its applications to nonlinear equations. Phys. Lett. A **277**, 212–218 (2000)
14. Cheemaa, N., Younis, M.: New and more exact traveling wave solutions to integrable (2+1)-dimensional Maccari system. Nonlinear Dyn. **83**, 1395–1401 (2016)
15. Bekir, A.: New exact travelling wave solutions of some complex nonlinear equations. Commun. Nonlinear Sci. Numer. Simul. **14**, 1069–1077 (2009)
16. Mirzazadeh, M., Eslami, M., Zerrad, E., Mahmood, M.F., Biswas, A., Belic, M.: Optical solitons in nonlinear directional couplers by sine–cosine function method and Bernoulli's equation approach. Nonlinear Dyn. **81**, 1933–1949 (2015)
17. Ren, Y.J., Zhang, H.Q.: A generalized F-expansion method to find abundant families of Jacobi elliptic function solutions of the (2 +1)-dimensional Nizhnik–Novikov–Veselov equation. Chaos Solitons Fractals **27**, 959–979 (2006)
18. Abdou, M.A.: Further improved F-expansion and new exact solutions for nonlinear evolution equations. Nonlinear Dyn. **52**, 277–288 (2008)
19. He, J.H., Wu, X.H.: Construction of solitary solution and compacton-like solution by variational iteration method. Chaos Solitons Fractals **29**(1), 108–113 (2006)
20. Bekir, A., Boz, A.: Application of He's exp-function method for nonlinear evolution equations. Comput. Math. Appl. **58**, 2286–2293 (2009)
21. He, J.H., Abdou, M.A.: New periodic solutions for nonlinear evolution equations using Exp-function method. Chaos Solitons Fractals **34**, 1421–1429 (2007)
22. Alam, MdN: Exact solutions to the foam drainage equation by using the new generalized (G'/G)-expansion method. Results Phys. **5**, 168–177 (2015)

23. Islam, MdS, Khan, K., Ali Akbar, M.: An analytical method for finding exact solutions of modified Korteweg-de Vries equation. Results Phys. **5**, 131–135 (2015)
24. Kaplan, M., Bekir, A., Ozer, M.N.: Solving nonlinear evolution equation system using two different methods. Open Phys. **13**, 383–388 (2015)
25. Inan, I.E., Ugurlu, Y., Inc, M.: New applications of the $(G\prime/G, 1/G)$-expansion method. Acta Phys. Pol. A **128**(3), 245–251 (2015)
26. Demiray, S., Unsal, O., Bekir, A.: New exact solutions for Boussinesq type equations by using $(G\prime/G, 1/G)$ and $(1/G\prime)$-expansion methods. Acta Phys. Pol. A **125**(5), 1093–1098 (2014)
27. Kaplan, M., Akbulut, A., Bekir, A.: Exact travelling wave solutions of the nonlinear evolution equations by auxiliary equation method. Zeitschrift für naturforschung A **70**(11), 969–974 (2015)
28. Abdou, M.A.: A generalized auxiliary equation method and its applications. Nonlinear Dyn. **52**, 95–102 (2008)
29. Adem, A.R., Khalique, C.M.: Conserved quantities and solutions of a (2+1)-dimensional Haragus-Courcelle–Il'ichev model. Comput. Math. Appl. **71**, 1129–1136 (2016)
30. Mirzazadeh, M., Arnous, A.H., Mahmood, M.F., Zerrad, E.: Soliton solutions to resonant nonlinear Schrödinger's equation with time-dependent coefficients by trial solution approach. Nonlinear Dyn. **81**, 277–282 (2015)
31. Bekir, A., Akbulut, A., Kaplan, M.: Exact solutions of nonlinear evolution equations by using modified simple equation method. Int. J. Nonlinear Sci. **19**(3), 159–164 (2015)
32. Akter, J., Akbar, M.A.: Exact solutions to the Benney–Luke equation and the Phi-4 equations by using modified simple equation method. Results Phys. **5**, 125–130 (2015)
33. Triki, H., Kara, A.H., Bhrawy, A.H., Biswas, A.: Soliton solution and conservation law of Gear–Grimshaw model for shallow water waves. Acta Phys. Pol. A **125**(5), 1099–1106 (2014)
34. Wang, G.-W., Xu, T.-Z., Abazari, R., Jovanoski, Z., Biswas, A.: Shock waves and other solutions to the Benjamin–Bona–Mahoney–Burgers equation with dual power-law nonlinearity. Acta Phys. Pol. A **126**(6), 1221–1225(2014)
35. Younis, M., Ali, S., Mahmood, S.A.: Solitons for compound KdV-Burgers equation with variable coefficients and power law nonlinearity. Nonlinear Dyn. **81**, 1191–1196 (2015)
36. Ali, S., Rizvi, S.T.R., Younis, M.: Traveling wave solutions for nonlinear dispersive water-wave systems with time-dependent coefficients. Nonlinear Dyn. **82**, 1755–1762 (2015)
37. Adem, A.R.: The generalized (1+1)-dimensional and (2+1)-dimensional Ito equations: multiple exp-function algorithm and multiple wave solutions. Comput. Math. Appl. **71**, 1248–1258 (2016)
38. Mirzazadeh, M., Eslami, M., Biswas, A.: 1-Soliton solution of KdV6 equation. Nonlinear Dyn. **80**, 387–396 (2015)
39. Islam, MdS, Khan, K., Arnous, A.H.: Generalized Kudryashov method for solving some (3+1)-dimensional nonlinear evolution equations. New Trends Math. Sci. **3**(3), 46–57 (2015)
40. Sanchez, P., Ebadi, G., Mojaver, A., Mirzazadeh, M., Eslami, M., Biswas, A.: Solitons and other solutions to perturbed Rosenau–KdV–RLW equation with power law nonlinearity. Acta Phys. Pol. A **127**(6), 1577–1586 (2015)
41. Hong-Cai, M.A., Zhen-Yun, Q.I.N., Ai-Ping, D.E.N.G.: Symmetry transformation and new exact multiple kink and singular kink solutions for (2+1)-dimensional nonlinear models generated by the Jaulent–Miodek hierarchy. Commun. Theor. Phys. **59**, 141–145 (2013)
42. Jawad, A.J.M., Mirzazadeh, M., Biswas, A.: Solitary wave solutions for nonlinear evolution equations in mathematical physics. Pramana J. Phys. **83**(4), 457–471 (2014)
43. Wazwaz, A.M.: Multiple-soliton solutions for the Calogero–Bogoyavlenskii–Schiff, Jimbo–Miwa and YTSF equations. Appl. Math. Comput. **203**, 592–597 (2008)
44. Moatimid, G.M., El-Shiekh, R.M., Al-Nowehy, A.G.A.: Exact solutions for Calogero–Bogoyavlenskii–Schiff equation using symmetry method. Appl. Math. Comput. **220**, 455–462 (2013)
45. Shakeel, M., Mohyud-Din, S.T.: Improved (G'/G)-expansion and extended tanh methods for (2+1)-dimensional Calogero-Bogoyavlenskii-Schiff equation. Alex. Eng. J. **54**, 27–33 (2015)