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Multiple-soliton solutions and analytical solutions to a nonlinear evolution equation

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Abstract The mathematical modelling of physical systems is generally expressed by nonlinear evolution equations. Therefore, it is critical to obtain solutions to these equations. We have employed the Hirota's method to derive multiple soliton solutions to (2+1)-dimensional nonlinear evolution equation. Then we have studied the transformed rational function method to construct different types of analytical solutions to the nonlinear evolution equations. This algorithm provides a more convenient and systematical handling of the solution process of nonlinear evolution equations, unifying the homogeneous balance method, the mapping method, the tanh-function method, the F-expansion method and the exp-function method.

Keywords Multiple-soliton solutions · Simplified Hirota's method · Exact solutions · Transformed rational function method

1 Introduction

Nonlinear evolution equations, which describe nonlinear phenomena, appear in a wide variety of applications in solitary waves theory, water waves, plasma physics, chemical physics, coastal engineering, hydrodynamics, theory of turbulence, optical fibers, fluid mechanics, chaos theory, ocean engineering, tsunami waves and many other applications. Recently, studies on these equations, integrable or non-integrable has made a substantial progress during the last few decades aims to shed light on features and properties of these

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equations. The soliton pulse, a perfect balance between nonlinearity and dispersion effects, recursion operators, the interaction between solitons, bi-Hamiltonian, symmetries, integrability, and other concepts are among many features which attracted many scientists to study (Chai et al. 2017; Liu et al. 2017).

It is significantly important to search for analytical solutions to nonlinear evolution equations. Exact solutions, including solitary wave solutions and soliton solutions, play a vital role in understanding various qualitative and quantitative features of nonlinear phenomena. Several powerful methods have been proposed to obtain exact solutions of these equations, such as the inverse scattering transform (Ablowitz and Clarkson 1991), the Hirota bilinear method (Hirota 1971), the Bäcklund transformation method (Li et al. 2002; Wang et al. 2015; Kaplan et al. 2017; Gao 2015, 2017, 2016), Lax pair (Yin et al. 2017), the Cole-Hopf transformation (Jordan 2010), Wronskian technique (Zhai and Chen 2008), the Miura transformation (Miura 1978), the Darboux transformation (Xue et al. 2012), the homogeneous balance method (Fan and Zhang 1998; Eslami et al. 2014), the Jacobi elliptic function method (Liu and Fan 2005), the homotopy analysis method (Kurt et al. 2016), similarity transformations method (Sahoo et al. 2017), the multiple exp-function method (Zayed and Al-Nowehy 2015), the linear superposition principle (Wang et al. 2011), the mapping method (Peng 2003), the first integral method (Zayed and Amer 2016; Mirzazadeh 2016), the Kudryashov method (Kaplan et al. 2016b), the trial equation method (Mirzazadeh 2015; Ekici et al. 2017; Zhou and Mirzazadeh 2016), the modified simple equation method (Mirzazadeh 2014; Akter 2015), the exp-function method (Fard et al. 2015), the $(G'/G)'$ expansion method (Mirzazadeh et al. 2017; Jawad et al. 2015) and so on Jabbari et al. (2011) Cheemaa and Younis (2016) Ebadi et al. (2012) and Zayed (2012).

Solitons are the most significant solutions among travelling wave solutions. The existence of multi-soliton, especially two-soliton solutions, is significant for information technology: it makes it possible undisturbed simultaneous propagation of many pulses in both directions.

The Hirota's method is as practical method to construct multiple-soliton solutions of nonlinear partial differential equations. It relies on a transformation for considered equation to a bilinear form. The bilinear forms are usually used to enable us deriving the auxiliary function. Note that finding such a transformation is not easy. Sometimes we need to introduce a new variable, dependent or independent. However, the simplified Hirota method is formally introduced the simplified algorithm to derive the auxiliary functions without using the bilinear forms (Wazwaz 2008; Alsayyeda et al. 2016).

Also a systematical and direct method, namely the transformed rational function method, was presented to construct analytical solutions of nonlinear evolution equations by use of rational function transformations. The keynote of this method is to find rational solutions to variable-coefficient ordinary differential equation transformed from considered nonlinear evolution equation. As what we have known, the transformed rational function method is systematic and unifies some of existing solution methods, such as the homogeneous balance method, F-expansion method, the tanh function method, the extended tanh function method and so on. Its key point is to find rational solutions to variable-coefficient ordinary differential equation transformed from given nonlinear partial differential equation (Ma and Lee 2009).

It is the aim of this work to derive multiple soliton solutions and different types of analytical solutions for the (2+1)-dimensional nonlinear evolution equation.

The Painleve test has been applied to the following (2+1)-dimensional nonlinear evolution equation (Bekir 2006):

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} = 0 \tag{1}$$

Then it is concluded that the equation has the Painleve property and also it is expected to be integrable. Also we use the extended homogenous balance method to shed light on the Auto-Bäcklund transformations of this equation (Kaplan et al. 2016a).

The rest of this paper is organized as follows. In Sect. 2, we derive the multiple-soliton solutions to Eq. (1) In Sect. 3, we give the description of the transformed rational function method. In Sect. 4, we apply this method to Eq. (1). Finally, some conclusions are given.

2 Solving the (2+1)-dimensional nonlinear evolution equation by the simplified Hirota's method

In this section, we obtain new soliton solutions of the Eq. (1). To derive the dispersion relation, we first substitute (Wazwaz and El-Tantawy 2016)

$$u(x, y, t) = e^{k_i x + l_i y - c_i t} \tag{2}$$

into the linear terms of Eq. (1) . Then we find as follows

$$c_i = \frac{k_i^3}{2}, \quad i = 1, 2, \ldots, N \tag{3}$$

and the dispersion variable becomes

$$\theta_i(x, y, t) = k_i x + l_i y - \frac{k_i^3}{2} t. \tag{4}$$

To obtain one-soliton solution, the following transformation is used

$$u(x, y, t) = R(\ln f(x, y, t))_x, \tag{5}$$

where $f(x, y, t)$ is given by

$$f(x, y, t) = 1 + e^{\theta_1} = 1 + e^{k_1 x + l_1 y - \frac{k_1^3}{2} t}. \tag{6}$$

Then we substitute Eqs. (5) into (1) and find

$$R = 2. \tag{7}$$

Therefore the single-soliton solution of Eq. (1) is founded as

$$u(x, y, t) = \frac{2k_1 e^{k_1 x + l_1 y - \frac{k_1^3}{2} t}}{1 + e^{k_1 x + l_1 y - \frac{k_1^3}{2} t}}. \tag{8}$$

Note that, this solution describes a shock wave. Then for the two soliton solution we substitute

$$f(x,y,t) = 1 + e^{\theta_1} + e^{\theta_2} + a_{12}e^{\theta_1+\theta_2} \tag{9}$$

$$= 1 + e^{k_1x+l_1y-\frac{k_1^3}{2}t} + e^{k_2x+l_2y-\frac{k_2^3}{2}t} + a_{12}e^{(k_1+k_2)x+(l_1+l_2)y-(\frac{k_1^3}{2}+\frac{k_2^3}{2})t} \tag{10}$$

By substituting Eqs. (9) and (5) into (1), and solving for the phase shift a_{12}, we find

$$a_{12} = \frac{(k_2-k_1)(l_2-l_1)}{(k_1+k_2)(l_1+l_2)} \tag{11}$$

and this can be generalized to

$$a_{ij} = \frac{(k_j-k_i)(l_j-l_i)}{(k_i+k_{2j})(l_i+l_j)}, 1 \le i < j \le N \tag{12}$$

For the three soliton solutions, we set

$$f(x,y,t) = 1 + e^{\theta_1} + e^{\theta_2} + e^{\theta_3} + a_{12}e^{\theta_1+\theta_2} \tag{13}$$

$$+\ a_{13}e^{\theta_1+\theta_3} + a_{23}e^{\theta_2+\theta_3} \tag{14}$$

$$+\ b_{123}e^{\theta_1+\theta_2+\theta_3} \tag{15}$$

Proceeding as before, we find

$$b_{123} = a_{12}a_{23}a_{13} \tag{16}$$

This shows that the three-soliton solutions are obtainable. The existence of three-soliton solutions often indicates the complete integrability of the equation under examination. The higher level soliton solutions, for $n \succeq 4$ can be obtained in a parallel manner. This confirms that the (2+1)-dimensional nonlinear evolution equation is completely integrable that admits multiple-soliton solutions of any order. At the same time, the integrability should be confirmed by other techniques such as the Lax pairs of the derived equations (Wazwaz and El-Tantawy 2016; Wazwaz 2016).

3 The transformed rational function method

In this section, we describe the of the transformed rational function method. For a given nonlinear differential equation, say, in two independent variables x, y and a dependent variable t (Zhang and Zhang 2011)

$$P(x,y,t,u_x,u_y,u_t,\ldots) = 0 \tag{17}$$

we use the following transformation:

$$u(x,y,t) = u(\xi), \xi = ax + by - \omega t, \tag{18}$$

where a, b and ω are constants. By substituting Eqs. (18) into (17), (17) is reduced to an ordinary differential equation (ODE) as follows

$$Q\left(x, y, t, u^{(r)}, u^{(r+1)}, \ldots\right) = 0, \tag{19}$$

where $u^{(i)} = \frac{d^i u}{d\xi^i}, i \geq 1$, and r is the least order of derivatives in the equation. We should integrate Eq. (19) with respect to ξ as soon as possible.

A crucial step of this method is to present a new variable $\eta = \eta(\xi)$, which satisfies a solvable ODE, for example, a first-order differential equation:

$$\eta' = F = F(\xi, \eta). \tag{20}$$

Here the prime denotes the differentiation with respect to ξ and F is a function of ξ and η. Two special solvable cases of Eq. (20) are:

$$F = F(\eta) = \eta, F = F(\eta) = \alpha + \eta^2, \alpha = constant \tag{21}$$

The corresponding first-order ODEs above have the particular solutions $\eta = e^{\xi}$ and

$$\eta(\xi) = \begin{cases} \sqrt{\alpha}\tan\sqrt{\alpha}\xi or - \sqrt{\alpha}\cot\sqrt{\alpha}\xi, \alpha > 0 \\ -\sqrt{-\alpha}\tanh\sqrt{-\alpha}\xi or - \sqrt{-\alpha}\coth\sqrt{-\alpha}\xi, \alpha < 0, \end{cases} \tag{22}$$

respectively. Those two cases correspond to the extended tanh-function and the exp-function method method, respectively.

More general assumptions than Eq. (21) gives different solutions to nonlinear wave equations. For example, if we choose $F^2(\eta) = k\eta^4 + l\eta^2 + m$ (k, l and m are constants), this yields the solutions obtained by extended F-expansion method.

We seek for the analytical solutions as rational functions

$$\upsilon(\eta) = \frac{p(\eta)}{q(\eta)} = \frac{p_n\eta^n + p_{n-1}\eta^{n-1} + \cdots + p_0}{q_s\eta^s + p_{s-1}\eta^{s-1} + \cdots + p_0}, (n, s \in), \tag{23}$$

where $p_i, 0 \leq i \preceq n$ and $q_i, 0 \leq i \preceq s$ are normally constants but could be functions of the independent variables as in the F-expansion method. The travelling solutions can be assumed as

$$u^{(r)}(\xi) = \upsilon(\eta) = \frac{p(\eta)}{q(\eta)}, \tag{24}$$

where $p(\eta)$ and $q(\eta)$ are polynomials as indicated above. It can be easily computed that

$$u^{(r+1)} = F\upsilon', u^{(r+2)} = F\partial_n u^{(r)} = F\upsilon' + F'\upsilon', \ldots, \tag{25}$$

which is based on $\partial_\xi = F\partial_\eta$. Note that by the prime, we denote the derivatives with respect to the involved variable, for instance, $u' = \frac{du}{d\xi}, \upsilon' = \frac{d\upsilon}{d\eta}$ and $\upsilon'' = \frac{d^2\upsilon}{d\eta^2}$.

It is supposed that the transformed equation (19) is a rational function equation of with a given pair of n and s. We can achieve for polynomial type nonlinear equations, when F is, for example, a rational function in η. Then we only need to force the numerator of the founded rational function in the transformed equation to be zero. This yields an algebraic equation system. Then we solve this system (which could be a differential system as in the F-expansion method) to find $p(\eta), q(\eta)$ and ξ. Finally, by integrating $\upsilon(\eta)$ with respect to ξ , r times, different types of analytical solutions can be obtained

$$u(x,y,t)=u(\xi)=\int\ldots\int\frac{p(\eta(\xi))}{q(\eta(\xi))}d\xi\ldots d\xi=\int_0^{\xi}\int_0^{\xi_r}\ldots\int_0^{\xi_2}\frac{p(\eta(\xi_1))}{q(\eta(\xi_1))}d\xi_1\ldots\xi_{r-1}\xi_r+\sum_{i=1}^{r}d_i\xi^{r-i}, \tag{26}$$

where $d_i, 1\leq i\leq r$ are arbitrary constants. If $r=1$, there is only the last definite integral over $[0,\xi]$ in (26). The founded analytical solutions will definitely contain a polynomial part in ξ when $r>1$.

The case of the extended tanh-function method

If we choose $\eta=-\tanh(\xi)$, then the solution function can be founded as

$$u(\xi)=\underbrace{\int\cdots\int}_{r}\frac{(-1)^n p_n\tanh^n(\xi)+(-1)^{n-1}p_{n-1}\tanh^{n-1}(\xi)+\cdots+p_0}{(-1)^s q_s\tanh s(\xi)+(-1)^{s-1}q_{s-1}\tanh^{s-1}(\xi)+\cdots+q_0}d\xi\ldots d\xi. \tag{27}$$

which yields solitary wave solutions. If we choose $\eta=\tan(\xi)$, then the solution function can be founded as

$$u(\xi)=\underbrace{\int\cdots\int}_{r}\frac{p_n\tan^n(\xi)+p_{n-1}\tan^{n-1}(\xi)+\cdots+p_0}{q_s\tan s(\xi)+q_{s-1}\tan^{s-1}(\xi)+\cdots+q_0}d\xi\ldots d\xi. \tag{28}$$

which yields periodic wave solutions. The other choices of $\eta=-\coth(\xi)$ $\eta=-\cot(\xi)$ gives similar type analytical solutions. Note that we choose $\alpha=1$ and a general nonzero value of α may lead to more general solutions. If υ is a polynomial and $r=0$, then u gives an travelling wave solution in the form of a finite series in $\tan\xi$ or $\tanh\xi$ It can be obtained by using the tanh-function method.

The case of the exp-function method

If we choose $\eta=e^{\xi}$, then the solution function can be founded as

$$u(\xi)=\underbrace{\int\cdots\int}_{r}\frac{p_n e^{n\xi}+p_{n-1}e^{(n-1)\xi}+\cdots+p_0}{q_s e^{s\xi}+q_{s-1}e^{(s-1)\xi}+\cdots+q_0}d\xi\ldots d\xi, \tag{29}$$

which yields solutions founded by use of exp-function method.

This method unifies the existing methods using tan functions, tanh functions, the Jacobi elliptic functions and the exponential functions. Also it allows us to perform the computation more conveniently and systematically with Maple packet programme.

4 Application of transformed rational function method

For obtaining travelling wave solutions of Eq. (1), we use the travelling wave transformation Eq. (18) to find

$$a^3bu^{(4)}+6a^2bu'u''-2b\omega u''=0. \tag{30}$$

Here $'$ denotes the differentiation with respect to ξ. Then by integrating Eq.(30) with respect to ξ once, we find

$$a^3bu''' + 3a^2b(u')^2 - 2b\omega u' = 0, \tag{31}$$

The lowest order derivative in the equation is $r = 1$. We set $u' = v$ and obtain the following transformed (2+1)-dimensional nonlinear evolution equation. Here the prime denotes the differentiation with respect to ξ and F is a function of ξ and η.

$$a^3bF^2\upsilon'' + a^3bFF'\upsilon' + 3a^2b\upsilon^2 - 2b\omega\upsilon = 0, \tag{32}$$

where $'$ denotes the differentiation with respect to η.

4.1 The case $\eta' = \eta$

In this case, the transformed (2+1)-dimensional nonlinear evolution equation (32) turns into

$$a^3b\eta^2\upsilon'' + a^3b\eta\upsilon' + 3a^2b\upsilon^2 - 2b\omega\upsilon = 0 \tag{33}$$

By setting $m = n = 3$, we try to find rational solutions of υ. With a direct calculation, by using Maple we can notice that we have two non-constant solutions for υ:

$$\upsilon(\eta) = \frac{4aq_1q_2\eta}{4q_2^2\eta^2 + 4q_1q_2\eta + q_1^2}, \omega = \frac{a^3}{2}, \tag{34}$$

and

$$\upsilon(\eta) = \frac{p_0(p_1^2\eta^2)}{q_0(p_1^2\eta^2 - 8p_0p_1\eta + 16p_0^2)}, a = -\frac{3p_0}{q_0}, \omega = \frac{27p_0^3}{2q_0^3}. \tag{35}$$

Therefore, we find the following solutions for (32) (2+1)-dimensional nonlinear evolution equation

$$u(x, y, t) = -\frac{2aq_1}{q_1 + 2q_2e^{\xi}} + d, \xi = ax + by - \frac{a^3}{2}t \tag{36}$$

and

$$u(x, y, t) = \frac{24p_0^2}{q_0(4p_0 - p_1e^{\xi})} + \frac{p_0}{q_0}\xi + d, \xi = -\frac{3p_0}{q_0}x + by - \frac{27p_0^3}{2q_0^3}t. \tag{37}$$

Here d is the integration constant. Also all the constants are arbitrary. We can notice that second class of the analytical solutions (37) contain a linear function of ξ.

4.2 The case $\eta' = \alpha + \eta^2$

In that case, the transformed (2+1)-dimensional nonlinear evolution equation (32) turns into

$$a^3b\eta^4v'' + 2a^3b\eta^3v' + 2a^3b\alpha\eta^2v'' + a^3b\alpha^2v'' + 2a^3b\alpha\eta v' + 3a^2bv^2 - 2b\omega\upsilon = 0 \tag{38}$$

By setting $m = 3$ and $n = 1$, we seek a solution in rational form for υ. Similarly, with a direct calculation and by using Maple, it can be seen that for υ, just we have two options as follows:

$$v(\eta) = -2a\alpha - 2a\eta^2, \omega = -2a^3\alpha, \tag{39}$$

and

$$v(\eta) = -\frac{2}{3}a\alpha - 2a\eta^2, \omega = 2a^3\alpha. \tag{40}$$

By setting the constants again, the value of α which is not equal to zero does not give more general solutions. So, by setting $\alpha = 1$ and taking $\eta = \tan\xi$, $\eta = -\cot\xi$ we obtain the travelling wave solutions of (32) (2+1)-dimensional nonlinear evolution equation

$$\begin{cases} u(x,y,t) = -2a\tan\xi + d, \xi = ax + by + 2a^3t, \\ u(x,y,t) = -2a\tan\xi + \frac{4a\xi}{3} + d, \xi = ax + by - 2a^3t, \end{cases} \tag{41}$$

and

$$\begin{cases} u(x,y,t) = 2a\cot\xi + d, \xi = ax + by + 2a^3t, \\ u(x,y,t) = 2a\cot\xi + \frac{4a\xi}{3} + d, \xi = ax + by - 2a^3t. \end{cases} \tag{42}$$

Here d is the integration constant. Here all the constants are arbitrary and the second solutions contains a linear function of ξ. It is obvious that, by setting $\eta = -\tanh\xi$ and $\eta = -\coth\xi$, we get the same solutions in the case $\eta' = \eta$.

5 Conclusion

The transformed rational function and simplified Hirota's method are applied successfully for solving the (2+1)-dimensional nonlinear evolution equation. The first method gives one-soliton, two-soliton and three-soliton solutions analytical solutions of this equation including exp-functions. The solutions includes some physical significant solutions such as soliton solutions, soliton-like solutions, rational solutions and other types of solutions. Analysis on the obtained soliton solutions shows that the soliton width and central frequency could be adjusted, and the soliton could be compressed or amplified through changing the linear and nonlinear dispersion effects. Furthermore, directions of the movement for the soliton central frequency could be adjusted. Secondly, we have obtained different types of analytical solutions to the (2+1)-dimensional nonlinear evolution equation. Finally to our knowledge these solutions are new and potentially useful for the soliton compression and amplification.

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