



Application of the modified Kudryashov method to the generalized Schrödinger–Boussinesq equations

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Abstract

In the paper, the modified Kudryashov method is applied to find new exact solutions for the generalized Schrödinger–Boussinesq equation with the help of symbolic computation package Maple through the complex transform. The obtained solutions have been checked by substituting back into its corresponding equation with the aid of Maple package program.

Keywords The generalized Schrödinger–Boussinesq equations · Modified Kudryashov method · Exact traveling wave solutions · Symbolic computation

1 Introduction

Recently, the studies of integrable or non-integrable nonlinear evolution equations with different nonlinearity have been popular in the area of mathematical physics such as plasma physics, fluid mechanics, physical chemistry, mathematical biology, optical fiber, the theory of turbulence, water waves and other applications. Analytical solutions of nonlinear partial differential equations (NPDEs) throw light on the concrete physical significance of any real-world problems. As an output, several researchers introduced a variety of powerful and robust analytical methods to find a special type of solutions of NPDEs with the help of Maple, Mathematica, and Matlab. The powerful and robust analytical methods are namely the Hirota method, the backlund transformation, G'/G -expansion method, $(G'/G, 1/G)$ -expansion method, the modified simple equation method, the auxiliary equation method, the sine–Gordon expansion method, the hyperbolic function method, new auxiliary equation method, an algebraic method, the extended trial equation method, extended simplest equation method and an improved generalized Jacobi elliptic function method (Li 2014; Chowdhury and Rao 1998; Huang 2013; Zayed and Alurfi 2014; Khater and Kumar 2017;

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Kaplan and Bekir 2016a, b; Kumar et al. 2017, 2018, b; Akbulut and Kaplan 2017; Bai 2001; Khater et al. 2017).

The present paper focus on the generalized Schrödinger–Boussinesq equations (Hon and Fan 2009; Gepreel 2016; Bilige et al. 2013; Ray 2017; Li et al. 2011) as follows:

$$\left. \begin{aligned} iE_t + E_{xx} + \alpha E &= NE \\ 3N_{tt} - N_{xxx} + 3(N^2)_{xx} + \beta N_{xx} &= (|E|^2)_{xx} \end{aligned} \right\} \quad (1)$$

where α and β are real parameters, $E(x, t)$ is a complex function which represents the complex Schrödinger field and $N(x, t)$ is a real function which represents the real Boussinesq field that raised in laser and plasma physics. Equation (1) is a known model that describe the various physical phenomena in laser and plasma, such as formation, langmuir field amplitude and intense electromagnetic waves and modulational instabilities (Li 2014; Chowdhury and Rao 1998; Hon and Fan 2009; Gepreel 2016; Bilige et al. 2013; Ray 2017; Li et al. 2011). Chanda and Chowdhury (1988) has been studied the complete integrability of the generalized Schrödinger–Boussinesq equations in the view of Painlevé analysis. An algebraic method is implemented to construct solitary wave solutions, Jacobi periodic wave solutions and a range of other solutions of physical interest by Hon and Fan (2009). Hon and Fan (2009) also demonstrates that the Jacobi elliptic periodic wave solutions exactly degenerate to the soliton solutions at a certain limit condition. Gepreel (2016) used the extended trial equation method to find some different types of exact solutions such as the Jacobi elliptic function solutions, soliton solutions, trigonometric function solutions and rational, exact solutions to the nonlinear coupled Schrödinger Boussinesq equations. Bilige et al. (2013) successfully constructed the new exact traveling wave solutions of the coupled Schrödinger–Boussinesq equation by utilizing the extended simplest equation method. The exact traveling wave solutions with double arbitrary parameters which have been expressed by the hyperbolic functions, the trigonometric functions and the rational functions respectively. Ray (2017) employed an improved algebraic method based on the generalized Jacobi elliptic function method with symbolic computation is used to construct more new exact solutions for coupled Schrödinger–Boussinesq equations. Li et al. (2011) applied an extended coupled sub-equations expansion method to look for new traveling wave solutions of the generalized Schrödinger–Boussinesq equations. Eslami (2015) studied the coupled Schrödinger–Boussinesq equation and new soliton-like solutions are obtained for this equation via the direct algebraic method under certain constraints on the coefficient functions. Some solitary wave solutions are derived from the hyperbolic function solutions for taken of special choice of parameter values. Neirameh (2015) received new traveling wave solutions of coupled Schrödinger–Boussinesq equation via simplest equation method. In this case, author discussed the derived new solutions of coupled Schrödinger–Boussinesq equation can be used to employ some practically physical and mechanical phenomena. Manafian and Aghdae (2016) executed the improved $\tan(\Phi(\xi)/2)$ -expansion method to seek the exact soliton solutions of the coupled Schrödinger–Boussinesq system. The exact particular solutions are of five types: exact soliton wave solution, exact periodic wave solution, exact singular kink-type wave solution, and exact singular cupson wave solution. Deng (2017) studied the coupled Schrödinger–Boussinesq equation for obtained solitary and periodic traveling wave solutions by using the method of dynamical systems. Yang et al. (2013) studied the asymptotic behavior of solutions for the discrete coupled nonlinear Schrödinger–Boussinesq equations. The authors first prove the existence of a global attractor for the generated semigroup and then obtain an upper bound of the Kolmogorov entropy for the obtained global attractor. Jiang and Dai (2013) found various closed-form

heteroclinic breather solutions including classical heteroclinic, heteroclinic breather and Akhmediev breather solutions for coupled Schrödinger–Boussinesq equation are obtained using two-soliton and homoclinic test methods. Hong and Lu (2013) used a general algebraic method based on the generalized Jacobi elliptic functions expansion method, the improved general mapping deformation method, and the extended auxiliary function method with computerized symbolic computation to construct more general and new exact solutions for coupled Schrödinger–Boussinesq equations. As a result, several families of new generalized Jacobi elliptic function wave solutions are obtained by using this method, some of them are degenerated to solitary wave solutions and trigonometric function solutions in the limited cases. Wang et al. (2016) adopted the method of dynamical system, the exact travelling wave solutions of the coupled nonlinear Schrödinger–Boussinesq equations are studied. Based on this method, the bounded exact travelling wave solutions are obtained which contain solitary wave solutions and periodic travelling wave solutions. The solitary wave solutions and periodic travelling wave solutions are expressed by the hyperbolic functions and the Jacobian elliptic functions.

The main aim of this study is to find the new exact solutions for the generalized Schrödinger–Boussinesq equations using the modified Kudryashov method. The modified Kudryashov method is one of the robust and effective method to find new exact solutions of nonlinear partial differential equations with integer and fractional order.

2 A brief description of modified Kudryashov method

First, we present a brief description of the modified Kudryashov method (Kumar et al. 2018a, b) to obtain the traveling wave solutions for a given general nonlinear partial differential equation (NPDE). Now, consider a general form of the nonlinear partial differential equation, say in two independent variables x and t , is given by

$$\left. \begin{aligned} P_1(E, N, E_t, N_t, E_x, N_x, E_{tt}, N_{tt}, E_{xx}, N_{xx}, \dots) &= 0 \\ P_2(E, N, E_t, N_t, E_x, N_x, E_{tt}, N_{tt}, E_{xx}, N_{xx}, \dots) &= 0 \end{aligned} \right\} \tag{2}$$

where $E = E(x, t)$ is a complex and $N = N(x, t)$ is a real unknown function, P_1 and P_2 are a polynomial in $E(x, t)$ and $N(x, t)$, and its derivative in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives. The main steps of modified Kudryashov approach is as follows:

Step 1 Implementing the transformation $E(x, t) = e^{i\theta} U(\xi)$, $N(x, t) = V(\xi)$, where $\theta = kx + \omega t$, $\xi = px + rt$, converts Eq. (2) to the following nonlinear ordinary differential equation

$$\left. \begin{aligned} Q_1(U, V, U', V', U'', V'', \dots) &= 0 \\ Q_2(U, V, U', V', U'', V'', \dots) &= 0 \end{aligned} \right\} \tag{3}$$

where Q_1 and Q_2 is a polynomial of U, V and its derivatives and the superscripts indicate the ordinary derivatives with respect to ξ .

Step 2 Let us assume that the solution $U(\xi)$ of the nonlinear Eq. (3) can be presented as

$$\left. \begin{aligned} U(\xi) &= a_0 + \sum_{i=1}^m a_i Q^i(\xi) \\ V(\xi) &= b_0 + \sum_{i=1}^n b_i Q^i(\xi) \end{aligned} \right\} \tag{4}$$

where the arbitrary constants $a_i (i = 1, 2 \dots, m)$ and $b_i (i = 1, 2 \dots, n)$ are determined later but $a_m \neq 0$ and $b_n \neq 0$ and m and n are a positive integer, which can be determined by using homogeneous balance principle on Eq. (3) and $Q(\xi)$ satisfies the following new ansatz equation

$$Q'(\xi) = (Q^2(\xi) - Q(\xi)) \ln a, \tag{5}$$

where $a \neq 0, 1$ and the general nontrivial solution of the Eq. (5) is $Q(\xi) = \frac{1}{1+da^\xi}$.

Step 3 Substituting Eq. (4) along with Eqs. (5) into (3) and equating the coefficients of powers of $Q^i(\xi)$ to zero, we get a system of algebraic equations in parameters $a_0, a_1, \dots, a_m, b_0, b_1, \dots, b_n, k$ and ω . The system of algebraic equations is solved by Maple, then the values of parameters can be obtained. By substituting the obtained the system of solutions into Eq. (4) along with general solution of Eq. (5), finally we explore a series of new exact travelling wave solutions for the nonlinear PDE (2).

3 Applications of the modified Kudryashov method

3.1 Complex transformation procedure for the generalized Schrödinger–Boussinesq equations

By considering the traveling wave transformation as:

$$E(x, t) = e^{i\theta} U(\xi), \quad N(x, t) = V(\xi), \quad \theta = kx + \omega t, \quad \xi = px + rt, \tag{6}$$

where k, ω, p and r are arbitrary constant to be determined.

$$\begin{aligned} iE_t &= -\omega e^{i\theta} U + ire^{i\theta} U', \quad E_{xx} = -k^2 e^{i\theta} U + i2pk e^{i\theta} U' \\ &+ p^2 e^{i\theta} U'', \quad N_{tt} = r^2 V'', \quad N_{xx} = p^2 V'', \\ N_{xxxx} &= p^4 V^{iv}, \quad (N^2)_{xx} \\ &= p^2 (V^2)'' \text{ and } (|E|^2)_{xx} = p^2 (U^2)'' \end{aligned}$$

Using the transformation Eqs. (6) into (1), we obtain the following system

$$(r + 2pk)U' = 0, \tag{7}$$

$$p^2 U'' - (\alpha - \omega - k^2)U - UV = 0, \tag{8}$$

$$(3r^2 + \beta p^2)V'' - p^4 V^{iv} + 3p^2 (V^2)'' - p^2 (U^2)'' = 0. \tag{9}$$

By setting the integration constant to zero and integrate two times with respect to ξ in Eq. (9), we find

$$(12k^2 + \beta)V - p^2 V'' + 3V^2 - U^2 = 0, \text{ with } r = -2pk. \tag{10}$$

Therefore, Eqs. (8) and (10) can be rearranged as the following after putting $r = -2pk$:

$$\left. \begin{aligned} p^2 U'' - (\alpha - \omega - k^2)U - UV &= 0 \\ (12k^2 + \beta)V - p^2 V'' + 3V^2 - U^2 &= 0 \end{aligned} \right\} \tag{11}$$

where the prime remarks the derivative with respect to ξ .

3.2 Exact solutions for the generalized Schrödinger–Boussinesq equations

In this section, the modified Kudryashov method will be executed to solve the generalized Schrödinger–Boussinesq equations. In this context, by balancing the highest order derivative terms and nonlinear terms in Eq. (11), yields $m = 2$ and $n = 2$. Now, we assume a formal solution of the Eq. (11) with the help of Eq. (4), obtain

$$\left. \begin{aligned} U(\xi) &= a_0 + a_1 Q(\xi) + a_2(Q(\xi))^2 \\ V(\xi) &= b_0 + b_1 Q(\xi) + b_2(Q(\xi))^2 \end{aligned} \right\} \tag{12}$$

By substituting Eq. (12) along with its second derivative into Eq. (11) and comparing the terms in the resulting equation, a nonlinear system is gained. By solving above nonlinear system using symbolic computation package, the following case is determined.

Set 1–1

$$\begin{aligned} a_0 &= 0, \quad a_1 = -6\sqrt{2}p^2(\ln a)^2, \quad a_2 = 6\sqrt{2}p^2(\ln a)^2, \quad b_0 = 0, \\ b_1 &= -6p^2(\ln a)^2, \quad b_2 = 6p^2(\ln a)^2, \\ k &= \pm \frac{1}{2} \sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)} \quad \text{and} \quad \omega = \frac{1}{12}(11p^2(\ln a)^2 + \beta) + \alpha. \end{aligned}$$

Now, the following new exact solutions for the generalized Schrödinger–Boussinesq equations are obtained:

$$\left. \begin{aligned} E_{1,2}(x, t) &= 6\sqrt{2}p^2(\ln a)^2 \times e^{i\theta} \left[-\frac{1}{da^{p\left(x\mp\sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)t}\right)}} + \frac{1}{\left(da^{p\left(x\mp\sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)t}\right)}\right)^2} \right] \\ N_{1,2}(x, t) &= -6p^2(\ln a)^2 \left[\frac{1}{da^{p\left(x\mp\sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)t}\right)}} - \frac{1}{\left(da^{p\left(x\mp\sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)t}\right)}\right)^2} \right] \end{aligned} \right\} \tag{13}$$

Set 1–2

$$\begin{aligned} a_0 &= 0, \quad a_1 = 6\sqrt{2}p^2(\ln a)^2, \quad a_2 = -6\sqrt{2}p^2(\ln a)^2, \quad b_0 = 0, \\ b_1 &= -6p^2(\ln a)^2, \quad b_2 = 6p^2(\ln a)^2, \quad k = \pm \frac{1}{2} \sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)} \quad \text{and} \\ \omega &= \frac{1}{12}(11p^2(\ln a)^2 + \beta) + \alpha. \end{aligned}$$

Now, the following new exact solutions for the generalized Schrödinger–Boussinesq equations are determined:

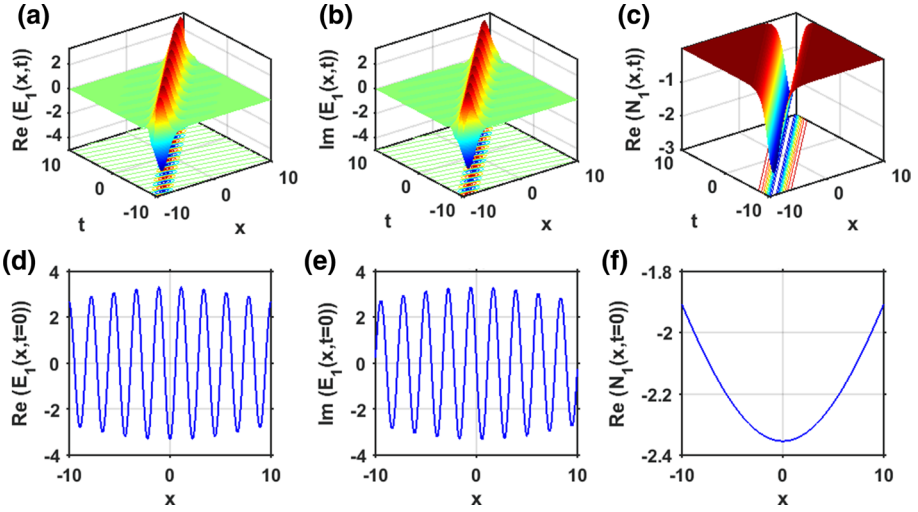


Fig. 1 a–c Three-dimensional (3D) plot with contours of Eq. (13) by choosing the value free parameters $a = 3.5$, $d = 1$, $\alpha = 1$, $\beta = -1$, and $p = 1$. d–e Two-dimensional (2D) plot of a–c respectively at $t = 0$

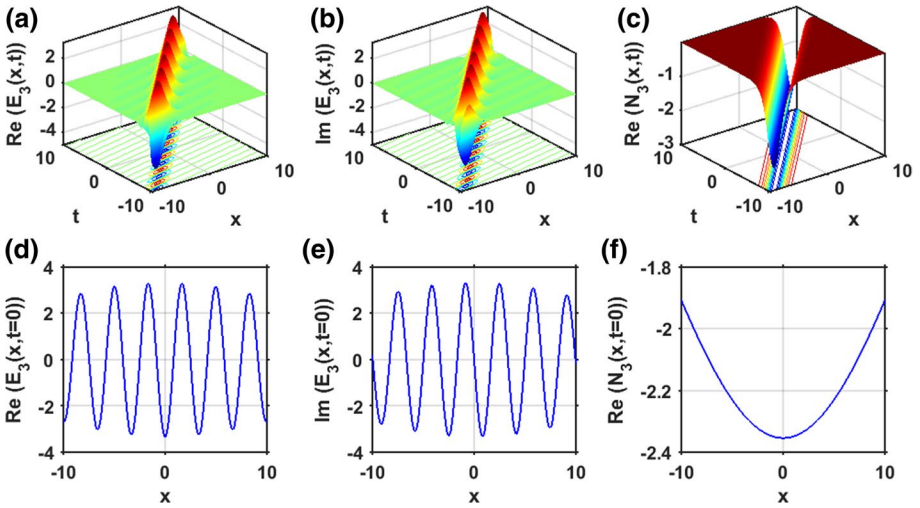


Fig. 2 a–c Three-dimensional (3D) plot with contours of Eq. (14) by choosing the value free parameters $a = 3.5$, $d = 1$, $\alpha = 1$, $\beta = -1$, and $p = 1$. d–e Two-dimensional (2D) plot of a–c respectively at $t = 0$

$$\left. \begin{aligned}
 E_{3,4}(x, t) &= 6\sqrt{2}p^2(\ln a)^2 \times e^{i\theta} \left(\frac{1}{da^{p\left(x\mp\sqrt{\frac{1}{3}(p^2(\ln a)^2-\beta)t}\right)}} - \frac{1}{\left(da^{p\left(x\mp\sqrt{\frac{1}{3}(p^2(\ln a)^2-\beta)t}\right)}\right)^2} \right) \\
 N_{3,4}(x, t) &= -6p^2(\ln a)^2 \left(\frac{1}{da^{p\left(x\mp\sqrt{\frac{1}{3}(p^2(\ln a)^2-\beta)t}\right)}} - \frac{1}{\left(da^{p\left(x\mp\sqrt{\frac{1}{3}(p^2(\ln a)^2-\beta)t}\right)}\right)^2} \right)
 \end{aligned} \right\} \quad (14)$$

For set 1–1 and set 1–2, $\theta = \pm\frac{1}{2}\sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)x} + \left(\frac{1}{12}(11p^2(\ln a)^2 + \beta) + \alpha\right)t$. Three-dimensional (3D) and two-dimensional (2D) plots of the Eqs. (13) and (14) is shown in Figs. 1 and 2.

Set 2–1

$$\begin{aligned}
 a_0 &= 0, \quad a_1 = -6\sqrt{2}p^2(\ln a)^2, \quad a_2 = 6\sqrt{2}p^2(\ln a)^2, \\
 b_0 &= \frac{1}{3}p^2(\ln a)^2, \quad b_1 = -6p^2(\ln a)^2, \quad b_2 = 6p^2(\ln a)^2, \\
 k &= \pm\frac{1}{2}\sqrt{-\frac{1}{3}(p^2(\ln a)^2 + \beta)} \text{ and } \omega = \frac{1}{12}(9p^2(\ln a)^2 + \beta) + \alpha.
 \end{aligned}$$

Now, the following new exact solutions for the generalized Schrödinger–Boussinesq equations are found:

$$\left. \begin{aligned}
 E_{5,6}(x, t) &= 6\sqrt{2}p^2(\ln a)^2 \times e^{i\theta} \left(-\frac{1}{da^{p\left(x\mp\sqrt{-\frac{1}{3}(p^2(\ln a)^2+\beta)t}\right)}} + \frac{1}{\left(da^{p\left(x\mp\sqrt{-\frac{1}{3}(p^2(\ln a)^2+\beta)t}\right)}\right)^2} \right) \\
 N_{5,6}(x, t) &= p^2(\ln a)^2 \left(\frac{1}{3} - \frac{6}{da^{p\left(x\mp\sqrt{-\frac{1}{3}(p^2(\ln a)^2+\beta)t}\right)}} + \frac{6}{\left(da^{p\left(x\mp\sqrt{-\frac{1}{3}(p^2(\ln a)^2+\beta)t}\right)}\right)^2} \right)
 \end{aligned} \right\} \quad (15)$$

Set 2–2

$$\begin{aligned}
 a_0 &= 0, \quad a_1 = 6\sqrt{2}p^2(\ln a)^2, \quad a_2 = -6\sqrt{2}p^2(\ln a)^2, \quad b_0 = \frac{1}{3}p^2(\ln a)^2, \\
 b_1 &= -6p^2(\ln a)^2, \quad b_2 = 6p^2(\ln a)^2, \\
 k &= \pm\frac{1}{2}\sqrt{-\frac{1}{3}(p^2(\ln a)^2 + \beta)} \text{ and } \omega = \frac{1}{12}(9p^2(\ln a)^2 + \beta) + \alpha.
 \end{aligned}$$

Now, the following new exact solutions for the generalized Schrödinger–Boussinesq equations are produced:

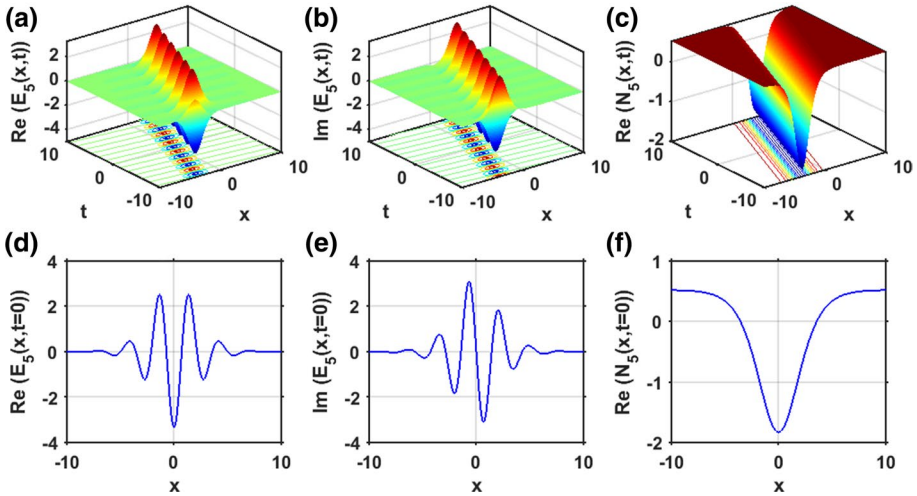


Fig. 3 a–c Three-dimensional (3D) plot with contours of Eq. (15) by choosing the value free parameters $a = 3.5$, $d = 1$, $\alpha = 1$, $\beta = -2$, and $p = 1$. d–e Two-dimensional (2D) plot of a–c respectively at $t = 0$

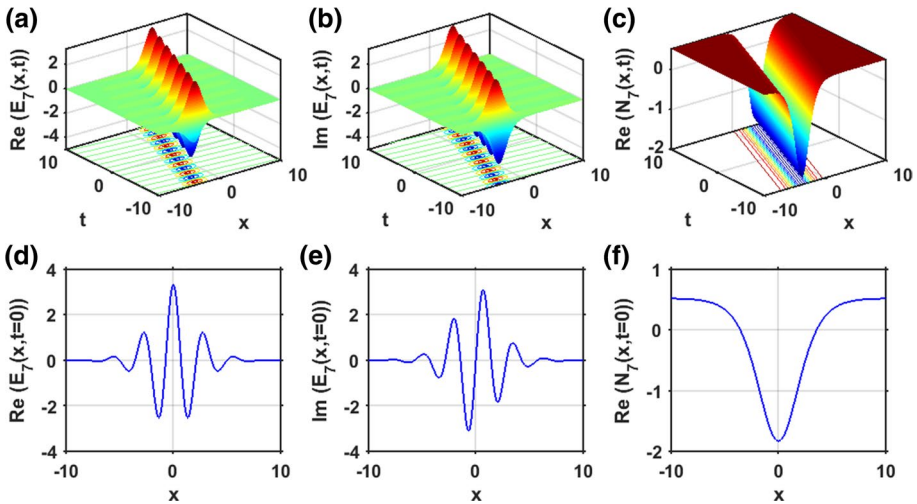


Fig. 4 a–c Three-dimensional (3D) plot with contours of Eq. (16) by choosing the value free parameters $a = 3.5$, $d = 1$, $\alpha = 1$, $\beta = -2$, and $p = 1$. d–e Two-dimensional (2D) plot of a–c respectively at $t = 0$

For set 2–1 and set 2–2, $\theta = \pm \frac{1}{2} \sqrt{-\frac{1}{3}(p^2(\ln a)^2 + \beta)}x + \left(\frac{1}{12}(9p^2(\ln a)^2 + \beta) + \alpha\right)t$.

Three-dimensional (3D) and two-dimensional (2D) plots of the Eqs. (15) and (16) is demonstrated in Figs. 3 and 4.

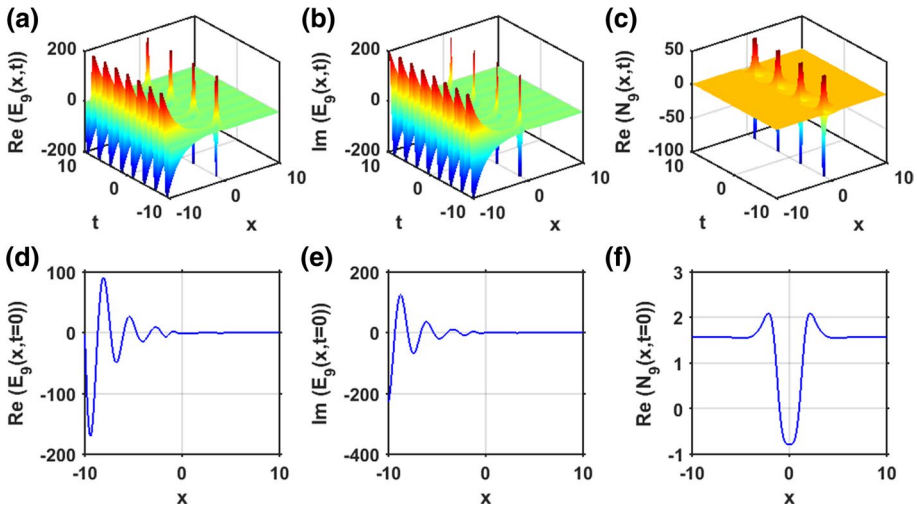


Fig. 5 a–c Three-dimensional (3D) plot of Eq. (17) by choosing the value free parameters $a = 3.5$, $d = 1$, $\alpha = 1$, $\beta = -1$, and $p = 1$. d–e Two-dimensional (2D) plot of a–c respectively at $t = 0$

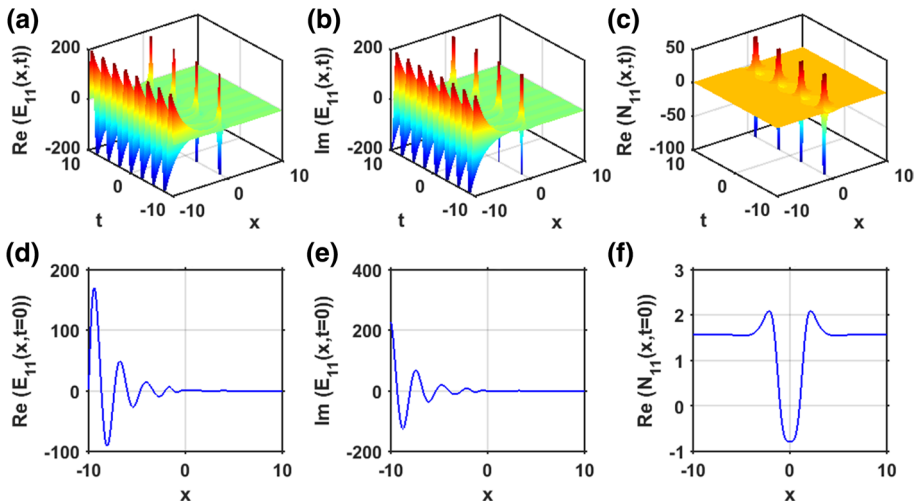


Fig. 6 a–c Three-dimensional (3D) plot of Eq. (18) by choosing the value free parameters $a = 3.5$, $d = 1$, $\alpha = 1$, $\beta = -1$, and $p = 1$. d–e Two-dimensional (2D) plot of a–c respectively at $t = 0$

Set 3–1

$$\begin{aligned}
 a_0 &= \sqrt{2}p^2(\ln a)^2, & a_1 &= -6\sqrt{2}p^2(\ln a)^2, & a_2 &= 6\sqrt{2}p^2(\ln a)^2, \\
 b_0 &= p^2(\ln a)^2, & b_1 &= -6p^2(\ln a)^2, & b_2 &= 6p^2(\ln a)^2, \\
 k &= \pm \frac{1}{2} \sqrt{-\frac{1}{3}(p^2(\ln a)^2 + \beta)} & \text{and } \omega &= -\frac{1}{12}(11p^2(\ln a)^2 - \beta) + \alpha.
 \end{aligned}$$

Now, the following new exact solutions for the generalized Schrödinger–Boussinesq equations are generated:

$$\left. \begin{aligned}
 E_{9,10}(x, t) &= \sqrt{2}p^2(\ln a)^2 \times e^{i\theta} \left(1 - \frac{6}{da^{p\left(x\mp\sqrt{-\frac{1}{3}(p^2(\ln a)^2+\beta)t}\right)}} + \frac{6}{\left(da^{p\left(x\mp\sqrt{-\frac{1}{3}(p^2(\ln a)^2+\beta)t}\right)}\right)^2} \right) \\
 N_{9,10}(x, t) &= p^2(\ln a)^2 \left(1 - \frac{6}{da^{p\left(x\mp\sqrt{-\frac{1}{3}(p^2(\ln a)^2+\beta)t}\right)}} + \frac{6}{\left(da^{p\left(x\mp\sqrt{-\frac{1}{3}(p^2(\ln a)^2+\beta)t}\right)}\right)^2} \right)
 \end{aligned} \right\} \quad (17)$$

Set 3–2

$$\begin{aligned}
 a_0 &= -\sqrt{2}p^2(\ln a)^2, \quad a_1 = 6\sqrt{2}p^2(\ln a)^2, \quad a_2 = -6\sqrt{2}p^2(\ln a)^2, \\
 b_0 &= p^2(\ln a)^2, \quad b_1 = -6p^2(\ln a)^2, \quad b_2 = 6p^2(\ln a)^2, \\
 k &= \pm\frac{1}{2}\sqrt{-\frac{1}{3}(p^2(\ln a)^2 + \beta)} \text{ and } \omega = -\frac{1}{12}(11p^2(\ln a)^2 - \beta) + \alpha.
 \end{aligned}$$

Now, the following new exact solutions for the generalized Schrödinger–Boussinesq equations are explored:

$$\left. \begin{aligned}
 E_{11,12}(x, t) &= \sqrt{2}p^2(\ln a)^2 \times e^{i\theta} \left(-1 + \frac{6}{da^{p\left(x\mp\sqrt{-\frac{1}{3}(p^2(\ln a)^2+\beta)t}\right)}} - \frac{6}{\left(da^{p\left(x\mp\sqrt{-\frac{1}{3}(p^2(\ln a)^2+\beta)t}\right)}\right)^2} \right) \\
 N_{11,12}(x, t) &= p^2(\ln a)^2 \left(1 - \frac{6}{da^{p\left(x\mp\sqrt{-\frac{1}{3}(p^2(\ln a)^2+\beta)t}\right)}} + \frac{6}{\left(da^{p\left(x\mp\sqrt{-\frac{1}{3}(p^2(\ln a)^2+\beta)t}\right)}\right)^2} \right)
 \end{aligned} \right\} \quad (18)$$

For set 3–1 and set 3–2, $\theta = \pm\frac{1}{2}\sqrt{-\frac{1}{3}(p^2(\ln a)^2 + \beta)x} + \left(-\frac{1}{12}(11p^2(\ln a)^2 - \beta) + \alpha\right)t$. Three-dimensional (3D) and two-dimensional (2D) plots of the Eqs. (17) and (18) is depicted in Figs. 5 and 6.

Set 4–1

$$\begin{aligned}
 a_0 &= \sqrt{2}p^2(\ln a)^2, \quad a_1 = -6\sqrt{2}p^2(\ln a)^2, \quad a_2 = 6\sqrt{2}p^2(\ln a)^2, \\
 b_0 &= \frac{2}{3}p^2(\ln a)^2, \quad b_1 = -6p^2(\ln a)^2, \quad b_2 = 6p^2(\ln a)^2, \\
 k &= \pm\frac{1}{2}\sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)} \text{ and } \omega = -\frac{1}{12}(9p^2(\ln a)^2 - \beta) + \alpha.
 \end{aligned}$$

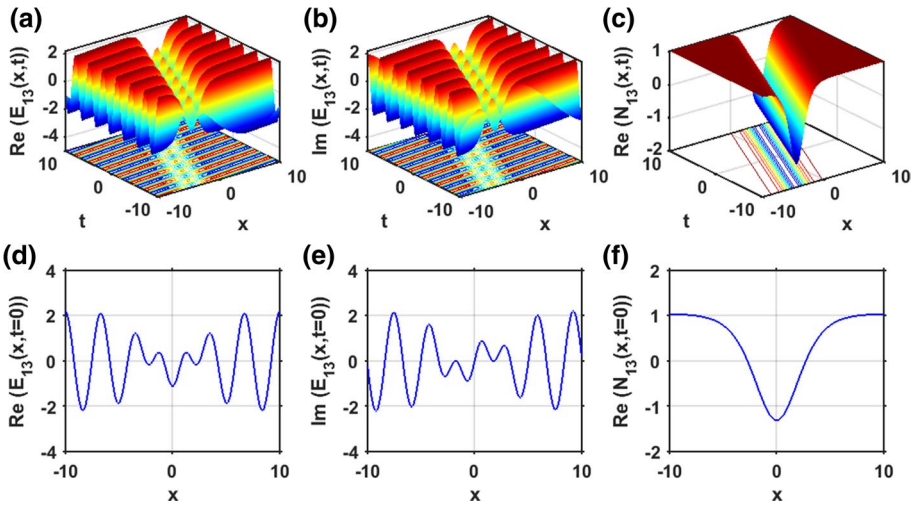


Fig. 7 a–c Three-dimensional (3D) plot with contours of Eq. (19) by choosing the value free parameters $a = 3.5$, $d = 1$, $\alpha = 1$, $\beta = -1$, and $p = 1$. d–e Two-dimensional (2D) plot of a–c respectively at $t = 0$

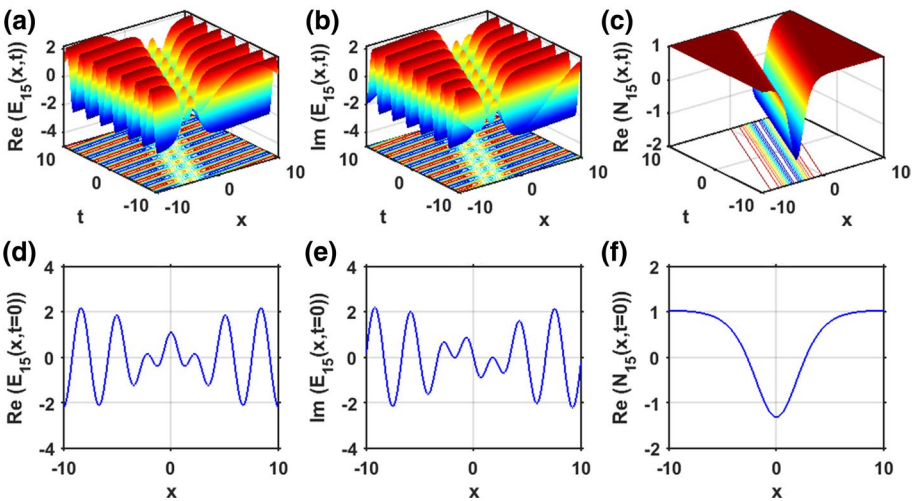


Fig. 8 a–c Three-dimensional (3D) plot with contours of Eq. (20) by choosing the value free parameters $a = 3.5$, $d = 1$, $\alpha = 1$, $\beta = -1$, and $p = 1$. d–e Two-dimensional (2D) plot of a–c respectively at $t = 0$

Now, the following new exact solutions for the generalized Schrödinger–Boussinesq equations are extracted:

$$\left. \begin{aligned}
 E_{13,14}(x, t) &= \sqrt{2}p^2(\ln a)^2 \times e^{i\theta} \left(1 - \frac{6}{da \left(x\mp \sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)t} \right)} + \frac{6}{\left(\frac{da \left(x\mp \sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)t} \right)} \right)^2} \right) \\
 N_{13,14}(x, t) &= p^2(\ln a)^2 \left(\frac{2}{3} - \frac{6}{da \left(x\mp \sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)t} \right)} + \frac{6}{\left(\frac{da \left(x\mp \sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)t} \right)} \right)^2} \right)
 \end{aligned} \right\} \tag{19}$$

Set 4–2

$$\begin{aligned}
 a_0 &= -\sqrt{2}p^2(\ln a)^2, \quad a_1 = 6\sqrt{2}p^2(\ln a)^2, \quad a_2 = -6\sqrt{2}p^2(\ln a)^2, \\
 b_0 &= \frac{2}{3}p^2(\ln a)^2, \quad b_1 = -6p^2(\ln a)^2, \quad b_2 = 6p^2(\ln a)^2, \\
 k &= \pm \frac{1}{2} \sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)} \text{ and } \omega = -\frac{1}{12}(9p^2(\ln a)^2 - \beta) + \alpha.
 \end{aligned}$$

Now, the following new exact solutions for the generalized Schrödinger–Boussinesq equations are obtained:

$$\left. \begin{aligned}
 E_{15,16}(x, t) &= \sqrt{2}p^2(\ln a)^2 \times e^{i\theta} \left(-1 + \frac{6}{da \left(x\mp \sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)t} \right)} - \frac{6}{\left(\frac{da \left(x\mp \sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)t} \right)} \right)^2} \right) \\
 N_{15,16}(x, t) &= p^2(\ln a)^2 \left(\frac{2}{3} - \frac{6}{da \left(x\mp \sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)t} \right)} + \frac{6}{\left(\frac{da \left(x\mp \sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)t} \right)} \right)^2} \right)
 \end{aligned} \right\} \tag{20}$$

For set 4–1 and set 4–2, $\theta = \pm \frac{1}{2} \sqrt{\frac{1}{3}(p^2(\ln a)^2 - \beta)}x + \left(-\frac{1}{12}(9p^2(\ln a)^2 - \beta) + \alpha \right)t$.

Three-dimensional (3D) and two-dimensional (2D) plots of the Eqs. (19) and (20) is shown in Figs. 7 and 8.

The obtained results are different from the existing ones in literature. Also the adopted algorithm is a direct and practical method.

4 Conclusion

The modified Kudryashov method was successfully utilized to establish a series of new traveling wave solutions for the generalized Schrödinger–Boussinesq equations. As a result, a series of new traveling wave solutions for the generalized Schrödinger–Boussinesq

equations were formally extracted. The performance of the modified Kudryashov method is reliable, effective and computerized mathematical tool to handle nonlinear evolution equations in the field of mathematical physics and applied sciences. The availability of computer systems like Maple and Mathematica facilitates the complicated and tedious algebraic calculations which can help to extract the new exact solutions for any nonlinear equations easily.

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