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The analysis of the soliton-type solutions of conformable equations by using generalized Kudryashov method

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Abstract

This research article is dedicated to applying the generalized Kudryashov method in order to acquire new exact and soliton-type solutions of the conformable Burgers' equation and Wu-Zhang system with conformable derivative.

Keywords : Exact solutions, generalized Kudryashov method, symbolic computation, conformable differential equations

MSC (2010) : 00A69, 26A33, 68W30

1 Introduction

In the literature, it is well known that a broad range of problems in many research fields, such as wave propagation phenomenon, dynamical systems, mechanical engineering, fluid mechanics, biology, hydrodynamics, plasma physics, image processing, chemistry, optics, finance, and other fields of engineering and science. Nonlinear fractional partial differential equations (NFPDEs) have been proposed and searched by several scientists [1, 2, 3]. Many proposed different definitions have introduced in the literature [4, 5, 6]. Recently, researchers started to see incompleteness in most of the fractional derivative definitions [7, 8].

Moreover, some functions do not have Taylor power series representation, or their Laplace transform can not calculate. Therefore, a new impressive definition called "the conformable derivative" was propounded by Khalil et al. in [9]. This depends just on the basic limit definition of the derivative. The conformable partial differential equations (CPDEs) are simply PDEs in sense of conformable partial fractional derivatives. Namely, conformable derivatives are easy to use while comparing to the other fractional derivatives, as its derivative definition does not include an integral term [28].

There are a lot of studies have done for the definition and properties of the conformable derivative. Conformable forms of the Taylor power series expansions, Gronwalls inequality, chain rule, exponential functions, Laplace transform, and integration by parts have been

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presented by T. Abdeljawad in [10]. N. Benkhettoua et al. [11] have been expressed the calculus of the conformable time-scale. Also, W. S. Chung [12] have used the conformable derivative and integral to work the fractional Newtonian mechanics. Moreover, the deterministic conformable partial differential equations (CPDEs) became a significant topic in the area. So, a lot of scientists paid more elaboration to their approximate and analytical solutions. The stochastic travelling wave solutions for the fractional coupled KdV and 2D KdV equations are acquired by a modified fractional sub-equation method in [13] and [14], respectively.

The generalized Kudryashov method is one of the most useful procedures to procure exact solutions of nonlinear partial differential equations [15, 16, 17].

This manuscript organized as follows. Firstly, the methodology of the generalized Kudryashov method has been given. Then, we implement the adopted method to the conformable Burgers' equation and Wu-Zhang system with conformable derivative for finding new exact travelling wave solutions. Also, we have plotted the graphics of the founded solutions have been given by setting some special values for the paremeters. Finally, a conclusion is given.

2 Summary of conformable fractional derivative

Assume that $f : [0, \infty) \to R$ be a function. The conformable derivative of f of order $\alpha, 0 < \alpha \leq 1$, is defined as

$$(T_\alpha f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon},$$

for all $t > 0$. We can list some helpful features as follows:

· $T_\alpha(af + bg) = a(T_\alpha f) + b(T_\alpha g)$, for all $a, b \in R$

· $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f)$

· Chain rule: Let $f : (0, \infty) \to R$ be a differentiable and $\alpha-$differerentiable function, g be a differentiable function defined in the range of f.

$$T_\alpha(f \circ g)(t) = t^{1-\alpha} g'(t) f'(g(t)).$$

Moreover, the following rules are hold.

$T_\alpha(t^p) = pt^{p-\alpha}$, for all $p \in R$

$T_\alpha(\lambda) = 0$, for all constant functions $f(t) = \lambda$

$T_\alpha(f/g) = \frac{g(T_\alpha f) - f(T_\alpha g)}{g^2}$.

Additively, if f is differentiable, then $T_\alpha(f)(t) = t^{1-\alpha} \frac{df}{dt}(t)$.[9, 18]

3 The generalized Kudryashov method

We take into consideration a general CPDE of the formula [19, 20, 21]:

$$F\left(u, \frac{\partial^\alpha u}{\partial t^\alpha}, \frac{\partial u}{\partial x}, \frac{\partial^{2\alpha} u}{\partial t^{2\alpha}}, \frac{\partial^2 u}{\partial x^2}, ...\right) = 0. \tag{1}$$

The polynomial and derivates of $u = u(x, t)$ are represented by F, where the nonlinear terms and the highest order derivatives are comprised. In a suitable manner of the following travelling wave transformation:

$$u(x, t) = u(\xi), \quad \xi = kx - l\frac{t^\alpha}{\alpha}, \tag{2}$$

where l represents the velocity of the wave, Eq. (1) is reduced thereby forming an ordinary differential equation (ODE) in the form

$$G(u, u', u'', ...,) = 0. \tag{3}$$

We note that, in Eq. (3) the differentiation of u with respect to ξ is represented by prime. We will integrate all the terms in Eq. (3). Conforming to this technique, the desired solution for the reduced equation is formed by a polynomial in $R(\xi)$ as

$$u(\xi) = \frac{\sum_{i=0}^{N} a_i R^i(\xi)}{\sum_{j=0}^{M} b_j R^j(\xi)}, \tag{4}$$

where $a_i (i = 0, 1, ..., n), b_j (j = 0, 1, ..., m)$ are constants to be found ($a_N \neq 0$, $b_M \neq 0$) and $Q = Q(\xi)$ is the solution of

$$\frac{dR}{d\xi} = R^2(\xi) - R(\xi). \tag{5}$$

The solution of Eq. (5) written as

$$R(\xi) = \frac{1}{1 + C_1 e^{\xi}}, \ C_1 \text{ is integration constant.}$$

Based on the homogeneous balance principle, one can find the positive integers N and M in Eq. (4) with the use of the (3). Finally, we can obtain a polynomial of R by subrogating Eq. (4) into Eq. (3) along with Eq. (5). Here, we equate all the coefficients of polynomial R to zero to obtain an algebraic equation system. Solutions of this system using the assistance of the computer software gives the values of $a_i (i = 0, 1, ..., n), b_j (j = 0, 1, ..., m)$. Lastly, we find the soliton-type solutions of the reduced Eq. (3) by subrogating these acquired values and Eq. (5) into Eq. (4).

4 Implementations

In the following section, two equations are implemented to show how the method works.

4.1 The conformable Burgers' equation

Burgers' equation

$$u_t + u u_x - v u_{xx} = 0,$$

was firstly presented in 1918 by Bateman has been used as a mathematical model in several fields such as gas dynamics, number theory, elasticity theory, heat conduction, hydrodynamic waves, shock wave theory, elastic waves, termaviscous fluids, and turbulence theory [22]. Many scientists have been worked on obtaining numerical and exact solutions for not only Burgers' equation [23, 24].

Then, the fractional Burgers' equation

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} + u u_x - v u_{xx} = 0, \ 0 < x < 1, t > 0, \tag{6}$$

where $\alpha \in (0,1)$ and $\dfrac{\partial^\alpha u}{\partial t^\alpha}$ means conformable derivative of $u(x,t)$ has attracted many scientists' attention. Using the Cole-Hopf transformation

$$u = -2v\frac{\delta_x}{\delta},$$

the equation Eq. (6) transforms into a time fractional heat equation,

$$\frac{\partial^\alpha \delta}{\partial t^\alpha} = v\frac{\partial^2 \delta}{\partial x^2} \tag{7}$$

where $\delta(x,t)$ the solution of the heat equation Eq. (7) and the derivative is an α-order conformable fractional derivative.

The numerical solution of fractional Burgers' equation has been acquired [25]. Esen [26] implemented HAM to procure the approximate analytical solution of fractional Burgers' equation. Also, exact solutions to this equation have been founded by using the Ricatti expansion method [27]. In 2015, Kurt et al. applied homotopy analysis method to this equation [28] and they utilized the Hopf-Cole transform [29]. Auto-Bäcklund transform and exact solutions to local conformable time-fractional viscous Burgers system have been found [30]. Also, some new travelling wave solutions for the one-dimensional Burgers equation was obtained [31].

Using the following travelling wave transformation

$$u(x,t) = \xi, \xi = x - c\frac{t^\alpha}{\alpha}, \tag{8}$$

we can reduce Eq. (6) to the following ODE

$$-cu' + uu' - vu'' = 0. \tag{9}$$

Integrating Eq. (9) with respect to ξ once, and taking the integration constant as zero, we find the following equation

$$-cu + \frac{u^2}{2} - vu' = 0. \tag{10}$$

According to the homogeneous balance principle, it can be founded as $N = M + 1$. By setting $M = 1$, we get $N = 2$. So the solution can be expressed as

$$u(\xi) = \frac{a_0 + a_1 R + a_2 R^2}{b_0 + b_1 R}, \tag{11}$$

where $R = R(\xi)$ is the solution of the Eq. (5). Based on this, we substitute Eq. (11) into Eq. (10) and use Eq. (5). Afterwards, we equate all coefficients of the functions R^k to zero. Therefore the following equation system can be obtained. Here a_0, a_1, a_2, b_0, and b_1 are parameters.

$$\begin{aligned}
R^4 &: \quad -va_2b_1 + \tfrac{a_2^2}{2} = 0,\\
R^3 &: \quad a_1a_2 - ca_2b_1 + va_2b_1 - 2va_2b_0 = 0,\\
R^2 &: \quad -ca_2b_0 + \tfrac{a_1^2}{2} + a_0a_2 + 2va_2b_0 - va_1b_0 + vb_1a_0 - ca_1b_1 = 0,\\
R^1 &: \quad -ca_0b_1 + va_1b_0 - ca_1b_0 + a_0a_1 - vb_1a_0 = 0,\\
R^0 &: \quad -ca_0b_0 + \tfrac{a_0^2}{2} = 0.
\end{aligned}$$

From the solution of this algebraic equation set, we find different cases which are discussed as follows.

Case 1:

$$a_0 = 0, a_1 = 0, a_2 = 2vb_1, b_0 = -\frac{b_1}{2}, c = 2v.$$

Then, by subrogating the acquisite values into Eq. (11) with Eq. (8), we get the soliton-type solutions of the conformable Burgers' equation as follows

$$u_1(x,t) = \frac{2vb_1\left(\frac{1}{1+C_1(\cosh(x-\frac{2vt^\alpha}{\alpha})+\sinh(x-\frac{2vt^\alpha}{\alpha}))}\right)^2}{-\frac{b_1}{2}+b_1\left(\frac{1}{1+C_1(\cosh(x-\frac{2vt^\alpha}{\alpha})+\sinh(x-\frac{2vt^\alpha}{\alpha}))}\right)}. \tag{12}$$

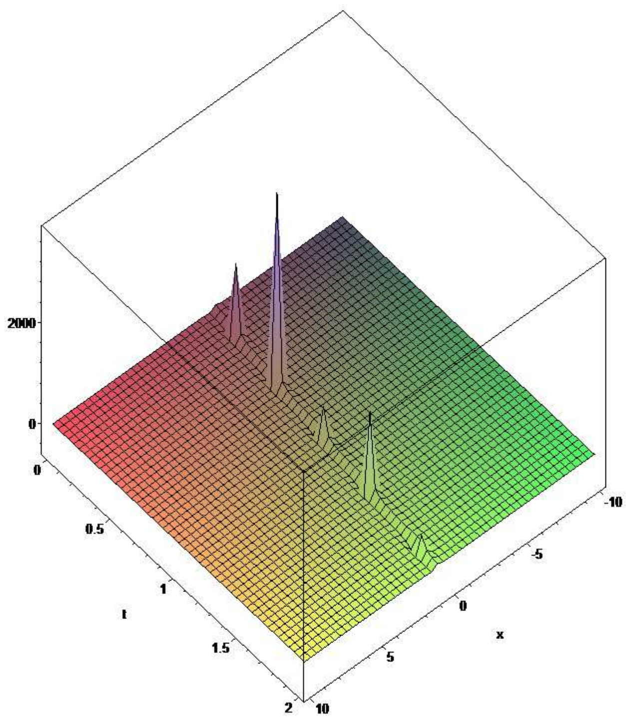


Figure 1 The figure of the solution for $u(x,t)$ obtained in Eq.(12) when $C_1 = 3, b_1 = 2, v = 4, \alpha = 0.9, c = 1$.

Case 2:

$$a_0 = 0, a_1 = 2vb_0, a_2 = 2vb_1, c = v.$$

Then, by subrogating the acquisite values into Eq. (11) with Eq. (8), we get the soliton-type solutions of the conformable Burgers' equation as follows

$$u_2(x,t) = \frac{2vb_0\left(\frac{1}{1+C_1(\cosh(x-\frac{vt^\alpha}{\alpha})+\sinh(x-\frac{vt^\alpha}{\alpha}))}\right)+2vb_1\left(\frac{1}{1+C_1e(\cosh(x-\frac{vt^\alpha}{\alpha})+\sinh(x-\frac{vt^\alpha}{\alpha}))}\right)^2}{b_0+b_1\left(\frac{1}{1+C_1(\cosh(x-\frac{vt^\alpha}{\alpha})+\sinh(x-\frac{vt^\alpha}{\alpha}))}\right)}. \tag{13}$$

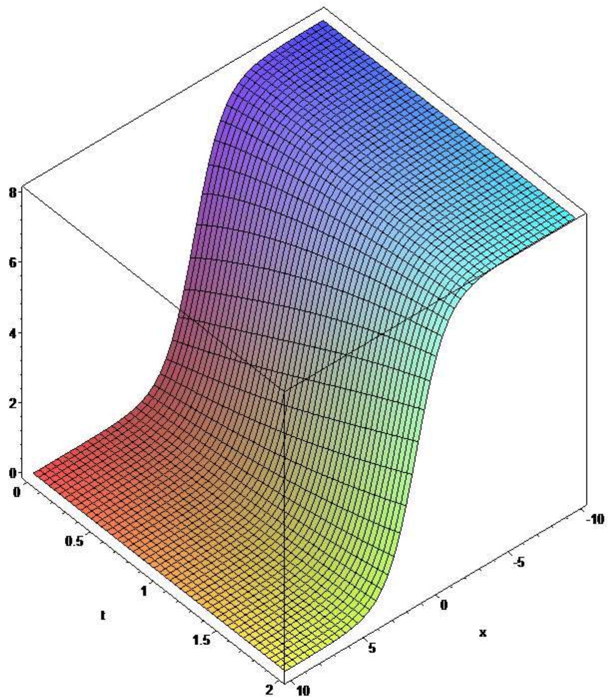


Figure 2 The figure of the solution for $u(x,t)$ obtained in $Eq.(13)$ when $C_1 = 3, b_0 = 1, b_1 = 2, v = 4, \alpha = 0.9, c = 1.$

Case 3:

$$a_0 = -2vb_0, a_1 = -2vb_1 + 2vb_0, a_2 = 2vb_1, c = -v.$$

Then, by subrogating the acquisite values into Eq. (11) with Eq. (8), we get the soliton-type solutions of the conformable Burgers' equation as follows

$$u_3(x,t) = \frac{-2vb_0 + (-2vb_1 + 2vb_0)\left(\frac{1}{1+C_1(\cosh(x+\frac{vt^{\alpha}}{\alpha})+\sinh(x+\frac{vt^{\alpha}}{\alpha}))}\right) + 2\alpha b_1\left(\frac{1}{1+C_1(\cosh(x+\frac{vt^{\alpha}}{\alpha})+\sinh(x+\frac{vt^{\alpha}}{\alpha}))}\right)^2}{b_0 + b_1\left(\frac{1}{1+C_1(\cosh(x+\frac{vt^{\alpha}}{\alpha})+\sinh(x+\frac{vt^{\alpha}}{\alpha}))}\right)}. \tag{14}$$

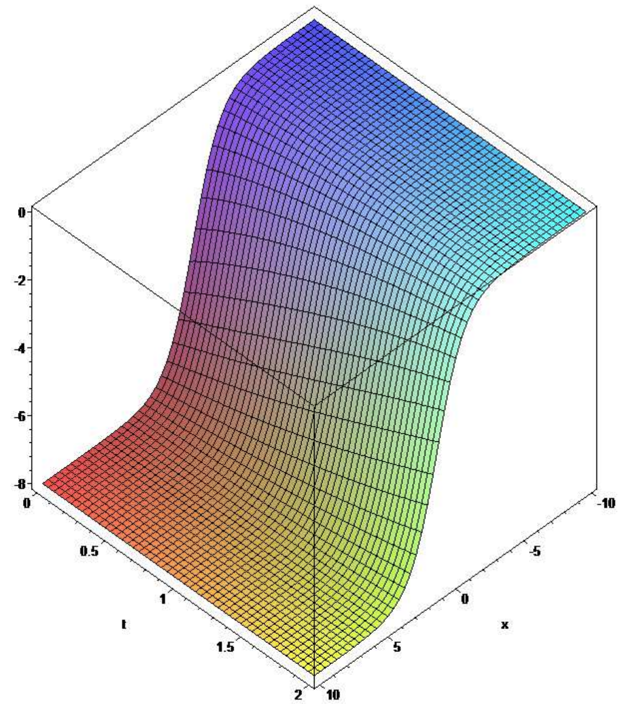


Figure 3 The figure of the solution for $u(x,t)$ obtained in Eq.(14) when $C_1 = 3, b_0 = 1, b_1 = 2, v = 4, \alpha = 0.9, c = 1$.

Case 4:

$$a_0 = 2vb_1, a_1 = -4vb_1, a_2 = 2vb_1, b_0 = -\frac{b_1}{2}, c = -2v.$$

Then, by subrogating the acquisite values into Eq. (11) with Eq. (8), we get the soliton-type solutions of the conformable Burgers' equation as follows

$$u_4(x,t) = \frac{2vb_1 - 4vb_1\left(\frac{1}{1+C_1(\cosh(x+\frac{2vt^\alpha}{\alpha})+\sinh(x+\frac{2vt^\alpha}{\alpha}))}\right) + 2vb_1\left(\frac{1}{1+C_1(\cosh(x+\frac{2vt^\alpha}{\alpha})+\sinh(x+\frac{2vt^\alpha}{\alpha}))}\right)^2}{-\frac{b_1}{2} + b_1\left(\frac{1}{1+C_1(\cosh(x+\frac{2vt^\alpha}{\alpha})+\sinh(x+\frac{2vt^\alpha}{\alpha}))}\right)}. \tag{15}$$

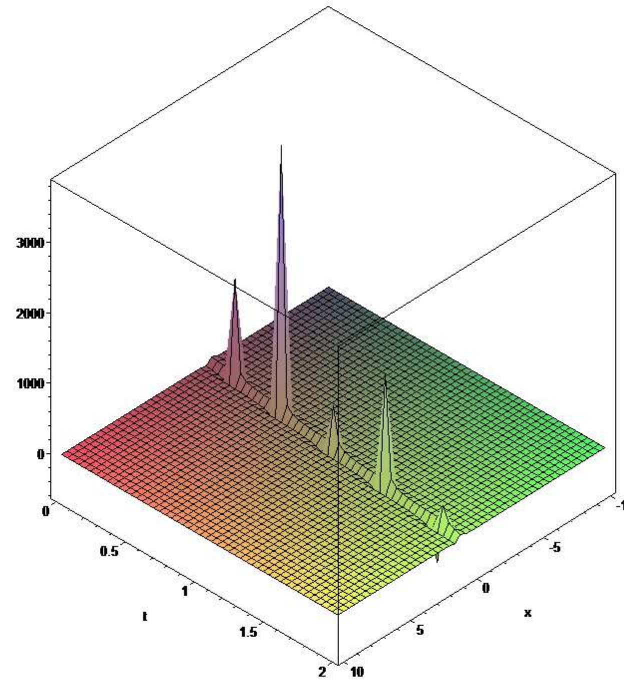


Figure 4 The figure of the solution for $u(x,t)$ obtained in Eq.(15) when $C_1 = 3, b_0 = 1, b_1 = 2, v = 4, \alpha = 0.9, c = 1$.

4.2 Wu-Zhang system with conformable derivative

Wu-Zhang system demonstrates dispersive long waves in two horizontal directions on shallow waters which means that the pure plane has several speeds propagation that makes some of the waves spread outwards in space. A proper comprehension of all solutions is useful for civil and coastal engineers in order to implement the nonlinear water wave model in the coastal and harbor design.

The (1+1)-dimensional Wu-Zhang system with conformable derivative is in the form [32]:

$$\begin{aligned} \frac{\partial^\alpha u}{\partial t^\alpha} + u\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} &= 0, \\ \frac{\partial^\alpha v}{\partial t^\alpha} + v\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial x} + \frac{1}{3}\frac{\partial^3 u}{\partial x^3} &= 0. \end{aligned} \tag{16}$$

Here u and v represent the elevation and the surface velocity of water, respectively.

In the year 1996, Zhang and Wu [33] proposed three sets of equations to model long, nonlinear, scattered gravitational waves that describe waves travelling in two horizontal directions over uniform shallow water depth. After doing some transformations and reductions, a dimensional long wave splitter (1+1) equation, known as the Wu-Zhang system has been acquired. These equations [34] are advantageous for harbor and coastal designs in civil and coastal engineering.

While there is a gap in the Wu- Zhang system's harmonious time zone literature, the classical Wu-Zhang system is thought to have achieved soliton solutions by many researchers [35, 36, 37]. In recent years, this system of equations has been discussed by many researchers to find exact solutions of different types. For example, Eslami and Rezazadeh are the first integral method applied to acquire exact solutions of Eq. (16) [32]. Khater et al. found numerical solutions through three different numerical schemes [38] and they found exact solutions through a modified auxiliary equation method applied for this system [39]. Yel and Baskonus [40] implemented the modified $exp(-\phi(\xi))$- expansion function method.

Let us to solve the system above based on the method implemented. We firstly take a travelling wave transformation as follows:

$$u(x,t) = u(\xi), v(x,t) = v(\xi), \xi = x - c\frac{t^{\alpha}}{\alpha}. \tag{17}$$

The sytem Eqs. (16) is reduced the following ODE with use of Eq. (17):

$$\begin{aligned} -cu' + uu' + v' &= 0, \\ -cv' + vu' + uv' + \tfrac{1}{3}u''' &= 0. \end{aligned} \tag{18}$$

Integrating first equation of Eqs. (18) with respect to ξ once and setting the constant of the integration to zero, we get the following equation:

$$v = cu - \frac{1}{2}u^2. \tag{19}$$

The following equation can be founded by subrogating Eq. (19) into the second equation of Eqs. (18):

$$-c^2u' + 3cuu' - \frac{3}{2}u^2u' + \frac{1}{3}u''' = 0. \tag{20}$$

If we integrate Eq. (20) with respect to ξ once, we find

$$u'' = 3c^2u - \frac{9}{2}cu^2 + \frac{3}{2}u^3. \tag{21}$$

Then we balance the highest order derivative term u'' with the nonlinear term u^3, we find $N = M + 1$. By setting $M = 1$, we find $N = 2$. Then, the desired exact solution becomes

$$u(\xi) = \frac{a_0 + a_1R + a_2R^2}{b_0 + b_1R}, \tag{22}$$

where $R = R(\xi)$ is the solution of the Eq. (5). Based on this, we get the following equation system by subrogating Eq. (22) into Eq. (21) along with Eq. (5) and then equating all coefficients of the functions R^k to zero:

$$\begin{aligned}
R^6 &: 2a_2b_1^2 - \tfrac{3}{2}a_2^3 = 0, \\
R^5 &: -3a_2b_1^2 + 6a_2b_0b_1 - \tfrac{9}{2}a_1a_2^2 + \tfrac{9}{2}ca_2^2b_1 = 0, \\
R^4 &: -\tfrac{9}{2}a_0a_2^2 + 9ca_1a_2b_1 + \tfrac{9}{2}ca_2^2b_0 - 3c^2a_2b_1^2 + 6a_2b_0^2 + a_2b_1^2 - 9a_2b_0b_1 - \tfrac{9}{2}a_1^2a_2 = 0, \\
R^3 &: -2b_1a_0b_0 + 3a_2b_0b_1 - 3c^2a_1b_1^2 + 9ca_0a_2b_1 - 9a_0a_1a_2 + a_1b_0b_1 - b_1^2a_0 \\
&\quad -6c^2a_2b_0b_1 - \tfrac{3}{2}a_1^3 - 10a_2b_0^2 + \tfrac{9}{2}ca_1^2b_1 + 9ca_1a_2b_0 + 2a_1b_0^2 = 0, \\
R^2 &: b_1^2a_0 - 3c^2a_2b_0^2 - \tfrac{9}{2}a_0a_1^2 - 3c^2a_0b_1^2 - \tfrac{9}{2}a_2a_0^2 - 3a_1b_0^2 - 6c^2a_1b_0b_1 + 9ca_0a_1b_1 \\
&\quad +3b_1a_0b_0 + 9ca_0a_2b_0 + \tfrac{9}{2}ca_1^2b_0 - a_1b_0b_1 + 4a_2b_0^2 = 0, \\
R^1 &: -6c^2a_0b_1b_0 - b_1a_0b_0 - \tfrac{9}{2}a_0^2a_1 + 9ca_0a_1b_0 + a_1b_0^2 - 3c^2a_1b_0^2 + \tfrac{9}{2}ca_0^2b_1 = 0, \\
R^0 &: \tfrac{9}{2}ca_0^2b_0 - 3c^2a_0b_0^2 - \tfrac{3}{2}a_0^3 = 0.
\end{aligned}$$

and a_0, a_1, a_2, b_0, b_1 are parameters to be found. Solving these algebraic equations with the assistance of Maple, we attain the following different cases:

Case 1:

$$a_0 = 0, a_1 = \mp\frac{2\sqrt{3}}{3}b_1, a_2 = \pm\frac{2\sqrt{3}}{3}b_1, b_0 = 0, c = \mp\frac{\sqrt{3}}{3}.$$

By subrogating the acquisite values into Eq. (22), we attain the soliton-type solutions of the system as

$$u_{1,2}(x,t) = \frac{\mp\frac{2\sqrt{3}}{3}b_1\left(\frac{1}{1+C_1\left(\cosh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})+\sinh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})\right)}\right) \pm \frac{2\sqrt{3}}{3}b_1\left(\frac{1}{1+C_1\left(\cosh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})+\sinh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})\right)}\right)^2}{b_1\left(\frac{1}{1+C_1\left(\cosh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})+\sinh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})\right)}\right)}, \tag{23}$$

and

$$v_{1,2}(x,t) = \frac{-8\left(1-2\left(\frac{1}{1+C_1\left(\cosh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})+\sinh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})\right)}\right)+\left(\frac{1}{1+C_1\left(\cosh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})+\sinh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})\right)}\right)^2\right)\left(\frac{1}{1+C_1\left(\cosh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})+\sinh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})\right)}\right)^2}{3\left(-1+2\left(\frac{1}{1+C_1\left(\cosh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})+\sinh(x\pm\frac{\sqrt{3}}{3\alpha}t^{\alpha})\right)}\right)\right)^2}. \tag{24}$$

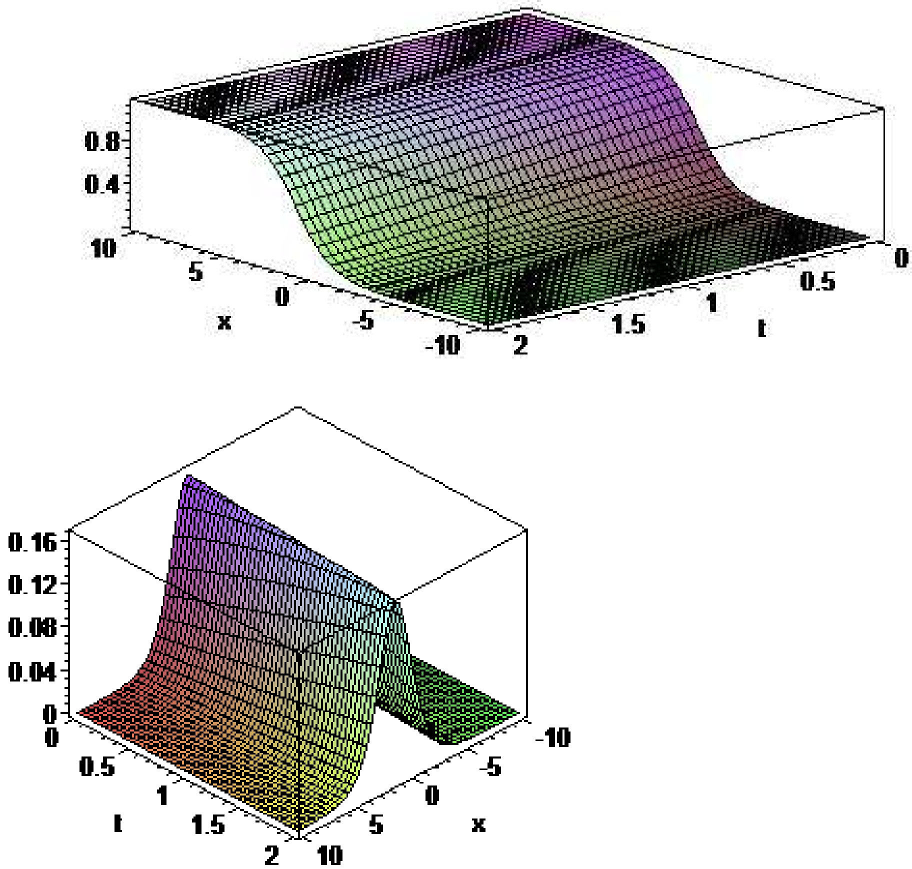


Figure 5 The figure of the solution for $u(x,t)$ and $v(x,t)$ obtained in Eq.(23) and Eq.(24) when $C_1 = 1, b_1 = 1, \alpha = 0.9$.

Case 2:

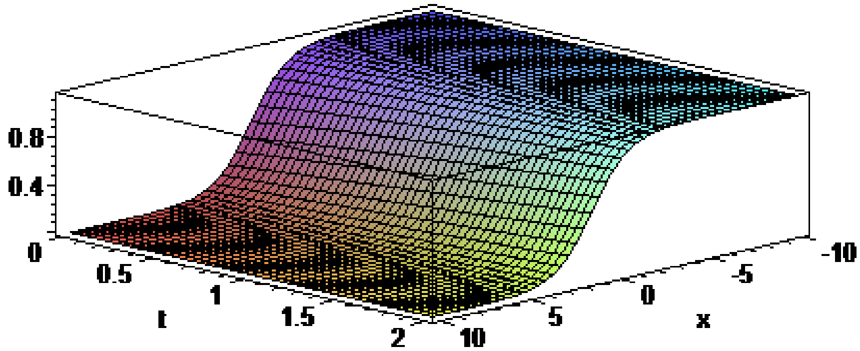
$$a_0 = 0, a_1 = \pm\frac{2\sqrt{3}}{3}b_0, a_2 = \pm\frac{2\sqrt{3}}{3}b_1, c = \pm\frac{\sqrt{3}}{3}.$$

Therefore, if we subrogate the acquisite values into Eq. (22), we acquire the soliton-type solutions of the Wu-Zhang system with conformable derivative as follows

$$u_{3,4}(x,t) = \frac{\pm\frac{2\sqrt{3}}{3}b_0\left(\frac{1}{1+C_1(\cosh(x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha)+\sinh(x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha))}\right) \pm \frac{2\sqrt{3}}{3}b_1\left(\frac{1}{1+C_1(\cosh(x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha)+\sinh(x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha))}\right)^2}{b_0 + b_1\left(\frac{1}{1+C_1(\cosh(x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha)+\sinh(x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha))}\right)}, \tag{25}$$

and

$$v_{3,4}(x,t) = -\frac{-2}{3\left(1 + C_1(\cosh(x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha) + \sinh(x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha))\right)}\left(-1 + \frac{1}{1 + C_1(\cosh(x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha) + \sinh(x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha))}\right). \tag{26}$$



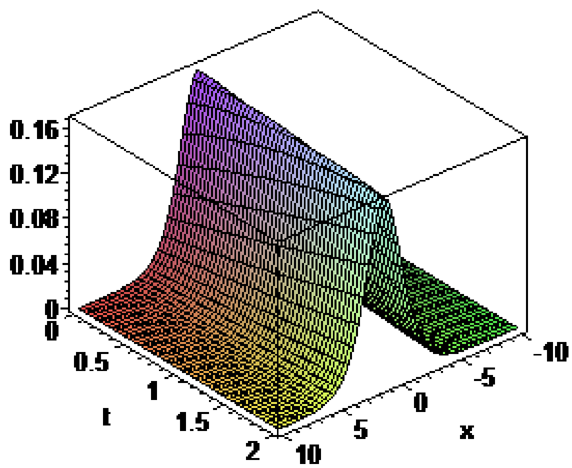


Figure.6 The figure of the solution for $u(x,t)$ and $v(x,t)$ obtained in Eq.(25) and Eq.(26) when $C_1 = 1, b_0 = 1, b_1 = 1, \alpha = 0.9$.

Case 3:

$$a_0 = 0, a_1 = 0, a_2 = \pm\frac{2\sqrt{3}}{3}b_1, b_0 = -\frac{b_1}{2}, c = \pm\frac{2\sqrt{3}}{3}.$$

By subrogating the acquisite values into Eq. (22), we acquire the soliton-type solutions of the Wu-Zhang system with conformable derivative as follows

$$u_{5,6}(x,t) = \frac{\pm\frac{2\sqrt{3}}{3}b_1\left(\frac{1}{1+C_1 e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^\alpha}}\right)^2}{-\frac{b_1}{2} + b_1\left(\frac{1}{1+C_1 e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^\alpha}}\right)}, \tag{27}$$

and

$$v_{5,6}(x,t) = \frac{-8\left(1 - 2\left(\frac{1}{1+C_1 e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^\alpha}}\right) + \left(\frac{1}{1+C_1 e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^\alpha}}\right)^2\right)\left(\frac{1}{1+C_1 e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^\alpha}}\right)^2}{3\left(-1 + 2\left(\frac{1}{1+C_1 e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^\alpha}}\right)\right)^2}. \tag{28}$$

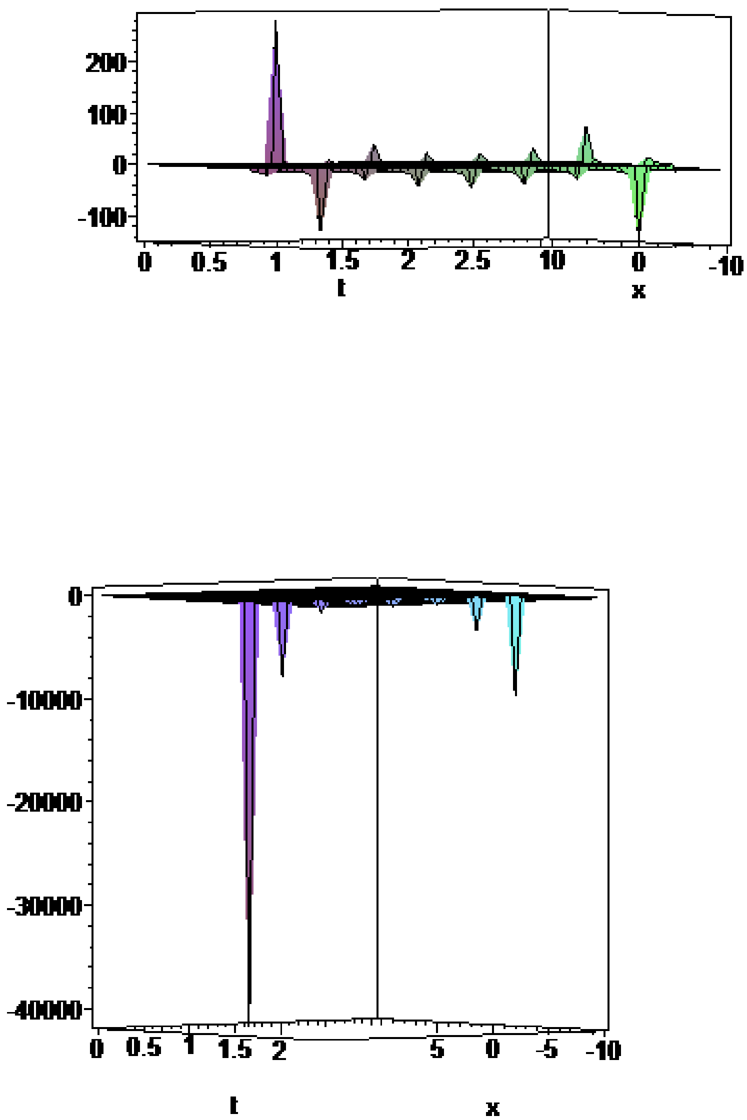


Figure 7 The figure of the solution for $u(x,t)$ and $v(x,t)$ obtained in Eq.(27) and Eq.(28) when $C_1 = 4, b_1 = 1, \alpha = 0.9$.

Case 4:

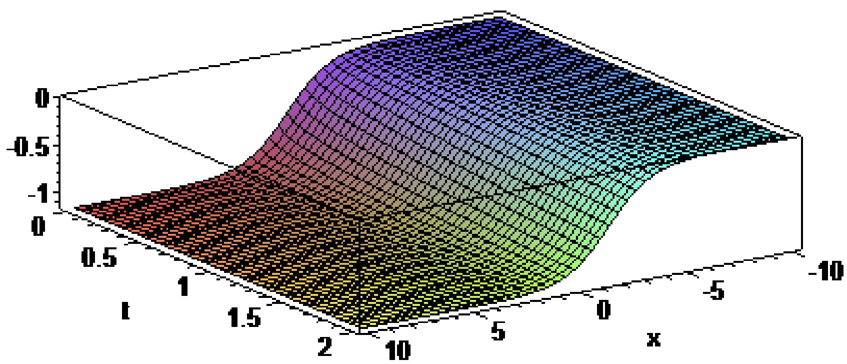
$$a_0 = \pm\frac{2\sqrt{3}}{3}b_0, a_1 = \mp\frac{2\sqrt{3}}{3}\left(-b_1 + b_0\right), a_2 = \mp\frac{2\sqrt{3}}{3}b_1, c = \pm\frac{\sqrt{3}}{3}.$$

By subrogating the acquisite values into Eq. (22), we acquire the soliton-type solutions of the Wu-Zhang system with conformable derivative as follows

$$u_{7,8}(x,t)=\frac{\pm\frac{2\sqrt{3}}{3}b_0\mp\frac{2\sqrt{3}}{3}(-b_1+b_0)\left(\frac{1}{1+C_1e^{x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha}}\right)\mp\frac{2\sqrt{3}}{3}b_1\left(\frac{1}{1+C_1e^{x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha}}\right)^2}{b_0+b_1\left(\frac{1}{1+C_1e^{x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha}}\right)},\tag{29}$$

and

$$v_{7,8}(x,t)=-\frac{-2}{3\left(\frac{1}{1+C_1e^{x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha}}\right)}\left(-1+\frac{1}{1+C_1e^{x\mp\frac{\sqrt{3}}{3\alpha}t^\alpha}}\right).\tag{30}$$



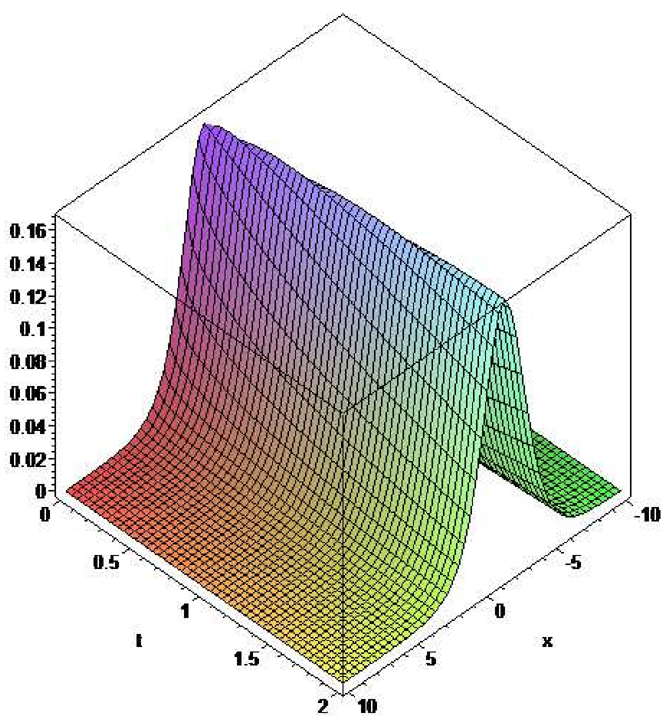


Figure 8 The figure of the solution for $u(x,t)$ and $v(x,t)$ obtained in Eq.(29) and Eq.(30) when $C_1=1, b_0=1, b_1=1, \alpha=0.9$.

Case 5:

$$a_0=\mp\frac{2\sqrt{3}}{3}b_1, a_1=\pm\frac{4\sqrt{3}}{3}b_1, a_2=\mp\frac{2\sqrt{3}}{3}b_1, b_0=-\frac{b_1}{2}, c=\pm\frac{2\sqrt{3}}{3}.$$

By subrogating the acquisite values into Eq. (22), we acquire the soliton-type solutions of the Wu-Zhang system with conformable derivative as follows

$$u_{9,10}(x,t)=\frac{\mp\frac{2\sqrt{3}}{3}b_1\pm\frac{4\sqrt{3}}{3}b_1\left(\frac{1}{1+C_1e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^{\alpha}}}\right)\mp\frac{2\sqrt{3}}{3}b_1\left(\frac{1}{1+C_1e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^{\alpha}}}\right)^2}{-\frac{b_1}{2}+b_1\left(\frac{1}{1+C_1e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^{\alpha}}}\right)}, \quad (31)$$

and

$$v_{9,10}(x,t)=\frac{-8\left(1-2\left(\frac{1}{1+C_1e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^{\alpha}}}\right)+\left(\frac{1}{1+C_1e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^{\alpha}}}\right)^2\right)\left(\frac{1}{1+C_1e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^{\alpha}}}\right)^2}{3\left(-1+2\left(\frac{1}{1+C_1e^{x\mp\frac{2\sqrt{3}}{3\alpha}t^{\alpha}}}\right)\right)^2}. \quad (32)$$

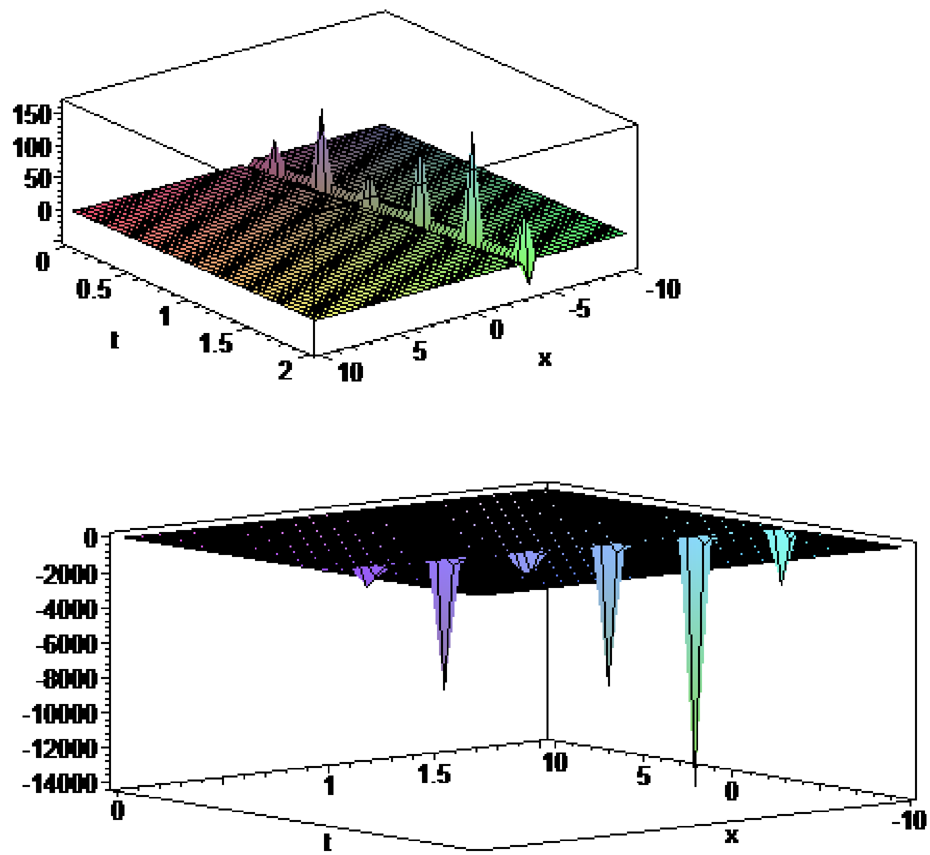


Figure 9 The figure of the solution for $u(x,t)$ and $v(x,t)$ obtained in Eq.(31) and Eq.(32) when $C_1=4, b_1=-1, \alpha=0.9$.

For all the cases above C_1 is an integration constant.

5 Conclusions

In this manuscript, the generalized Kudryashov method has been applied to acquire new soliton-type solutions of the conformable Burgers' equation and Wu-Zhang system with conformable derivative. By using the conformable fractional derivative definition in the travelling wave transformation, the equations are reduced to nonlinear ordinary differential equations. The acquired solutions represented by the exponential, hyperbolic trigonometric, and rational functions may be appropriate to understand the mechanism of the complex nonlinear physical phenomena in wave propagation. The obtained results are soliton-type results. Also, we have plotted the graphs of the acquired solutions under suitable values of constants have been given. By comparing our results with the existing ones in the literature, we saw that they are different. According to our knowledge, the obtained solutions in this article have the potential to shed light on the area of engineering.

The obtained solutions consisted of hyperbolic functions. These functions are circular functions that have arisen in both mathematics and physics. For instance, the hyperbolic secant functions arise in the profile of a laminar jet, the hyperbolic cosine functions have catenary shape, the hyperbolic tangent functions arise during computations of magnetic moment and rapidity of special relativity, and the hyperbolic cotangent functions arise in the Langevin function for magnetic polarisation.

The generalized Kudryashov method is an effective and powerful technique for acquiring soliton-type solutions of CPDEs. Also, it is deserving to study the Burgers' equation and Wu-Zhang system with other sense of fractional derivatives.

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Figures

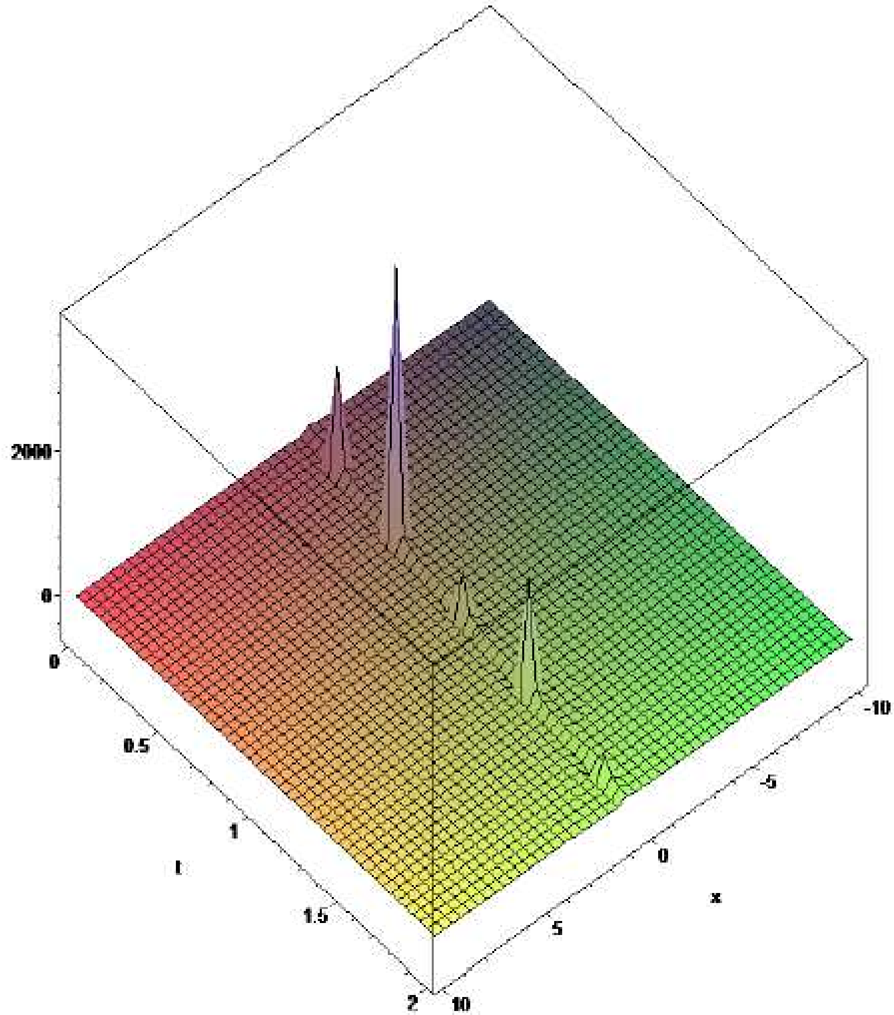

Figure 1

The figure of the solution for u(x; t) obtained in Eq.(12) when C1= 3; b1= 2; v = 4; α = 0:9; c = 1:

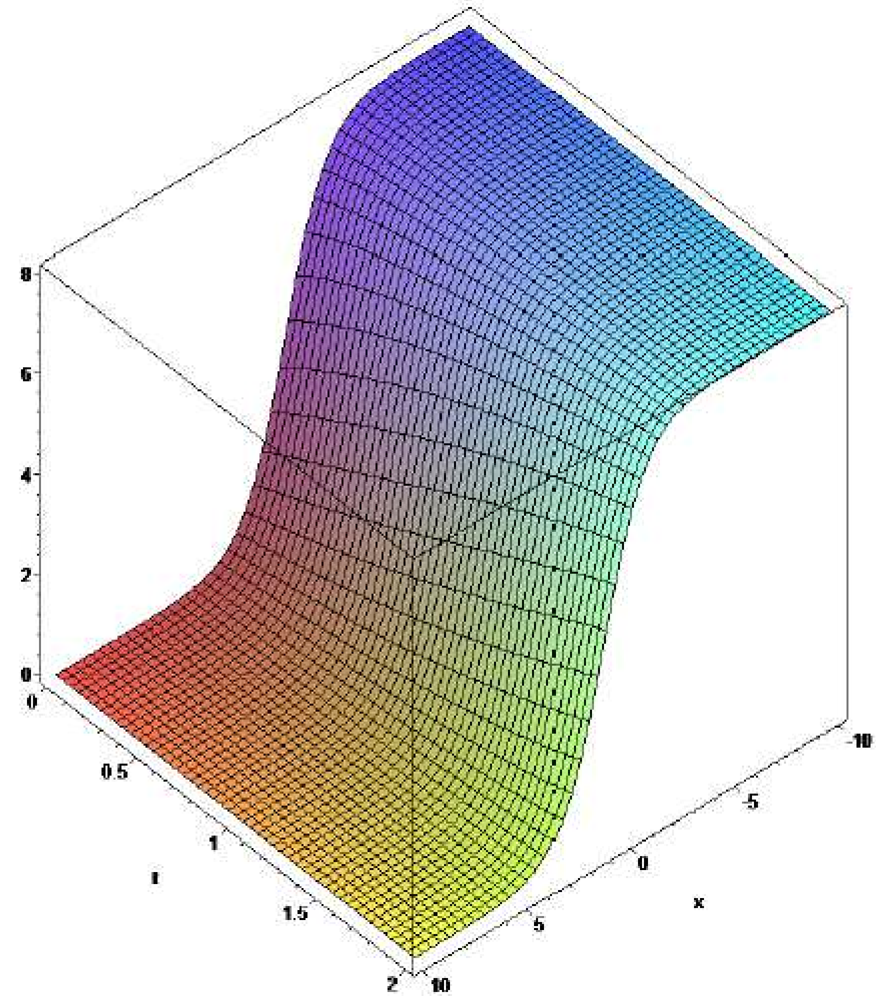

Figure 2

The figure of the solution for u(x; t) obtained in Eq:(13) when C1= 3; b0= 1; b1= 2; v = 4; α = 0:9; c = 1:

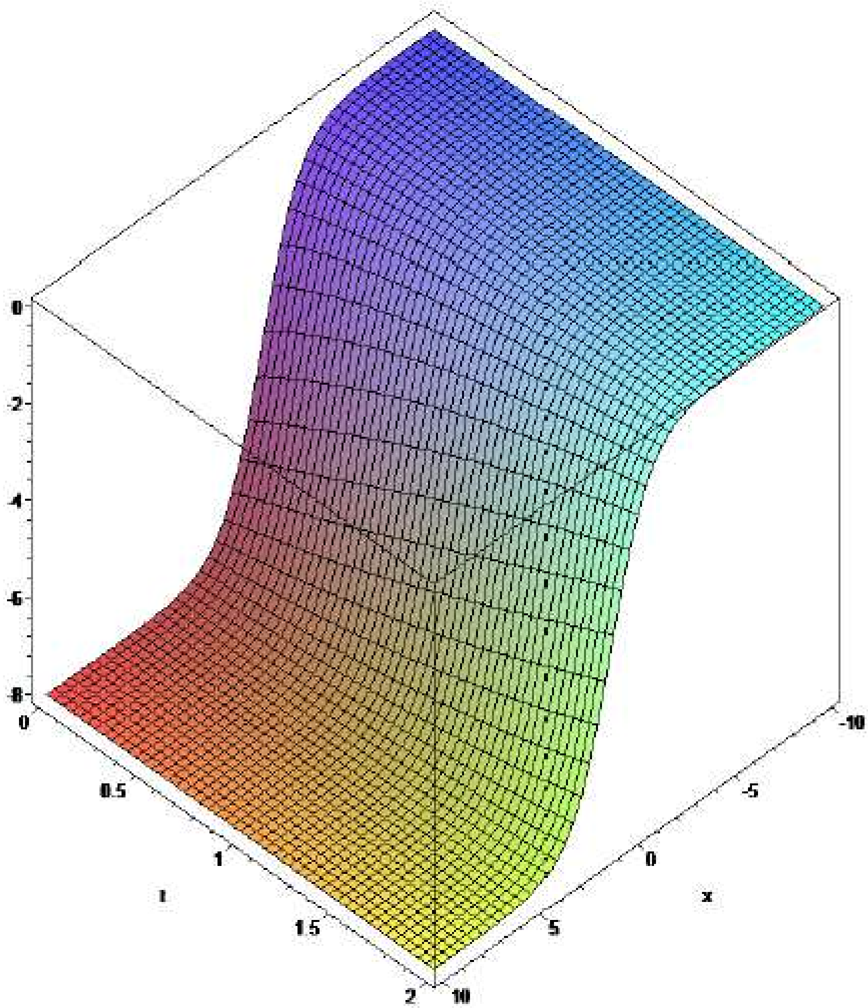

Figure 3

The figure of the solution for u(x; t) obtained in Eq.(14) when C1= 3; b0= 1; b1= 2; v = 4; α = 0:9; c = 1:

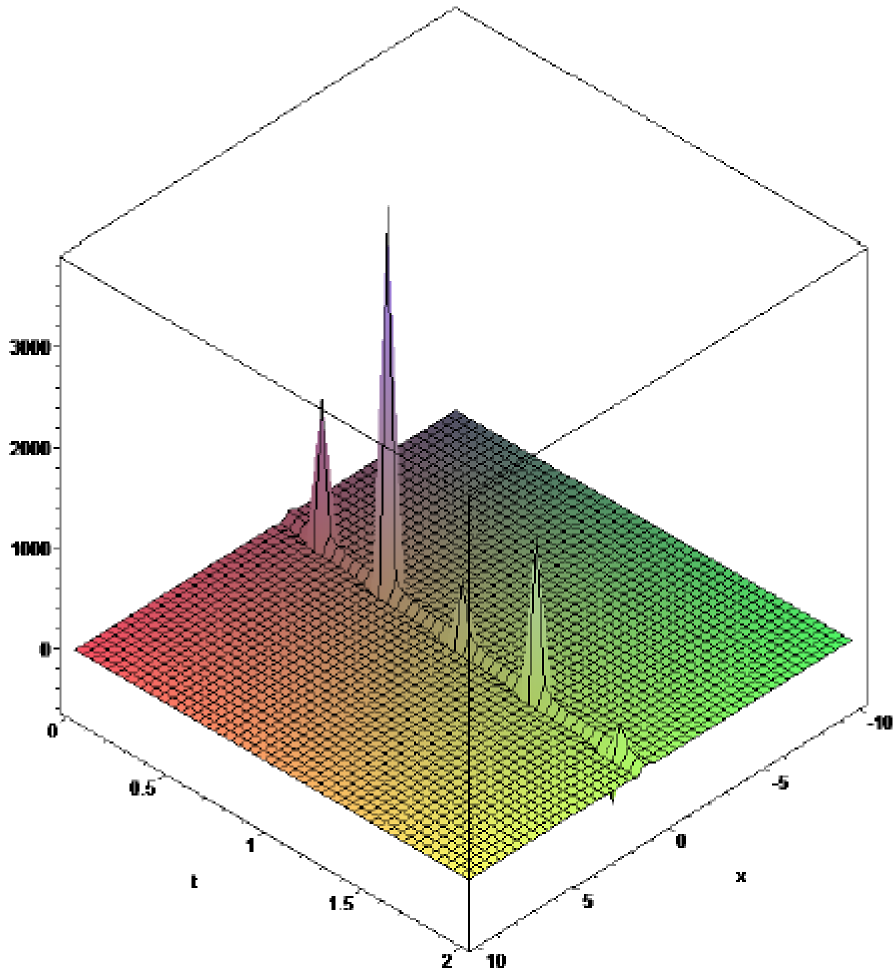

Figure 4

The figure of the solution for u(x; t) obtained in Eq.(15) when C1= 3; b0= 1; b1= 2; v = 4; α = 0:9; c = 1:

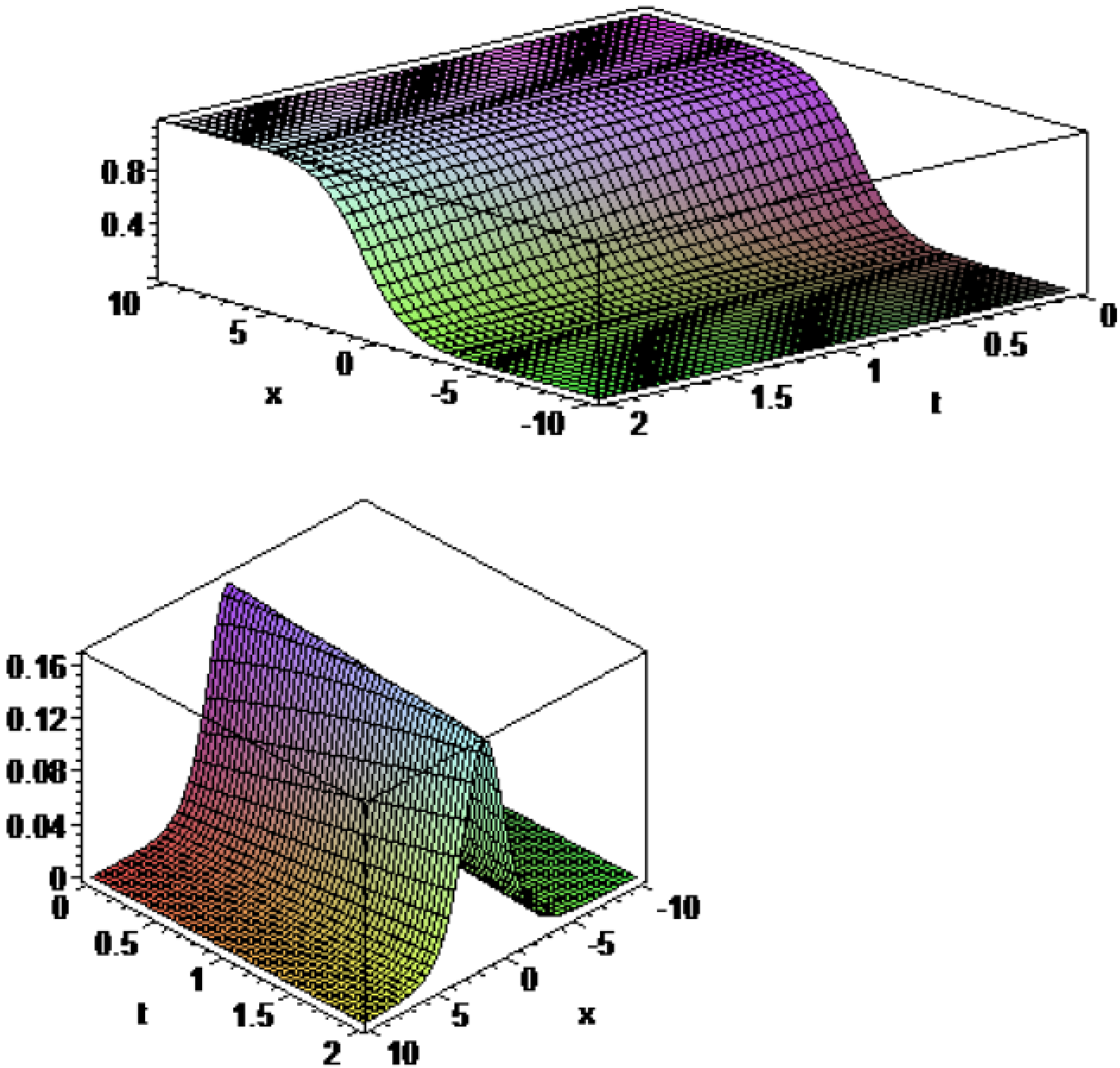

Figure 5

The figure of the solution for u(x; t) and v(x; t) obtained in Eq.(23) and Eq.(24) when C1= 1; b1= 1; α = 0:9:

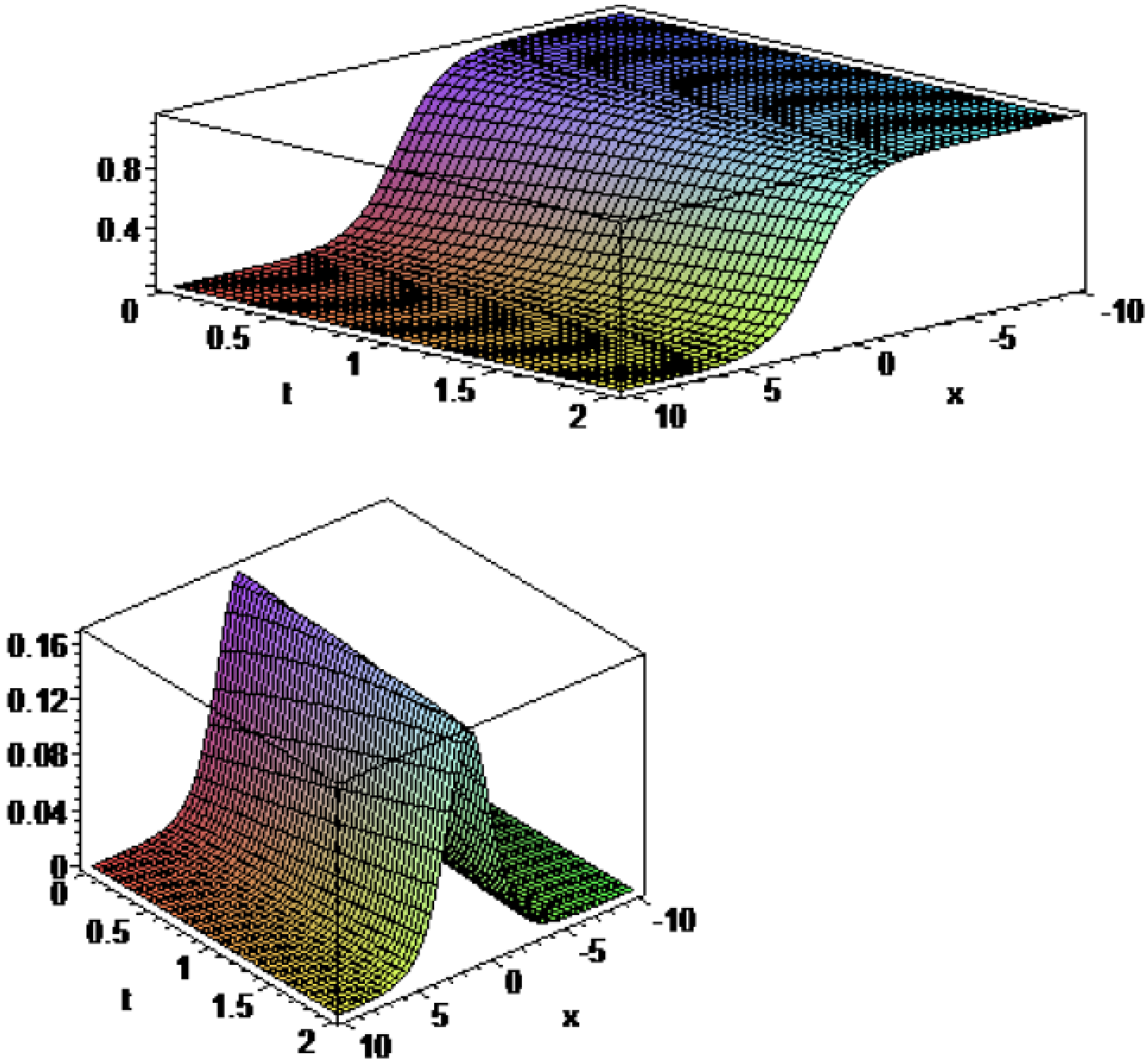

Figure 6

The figure of the solution for u(x; t) and v(x; t) obtained in Eq.(25) and Eq.(26) when C1= 1; b0= 1; b1= 1; α = 0:9:

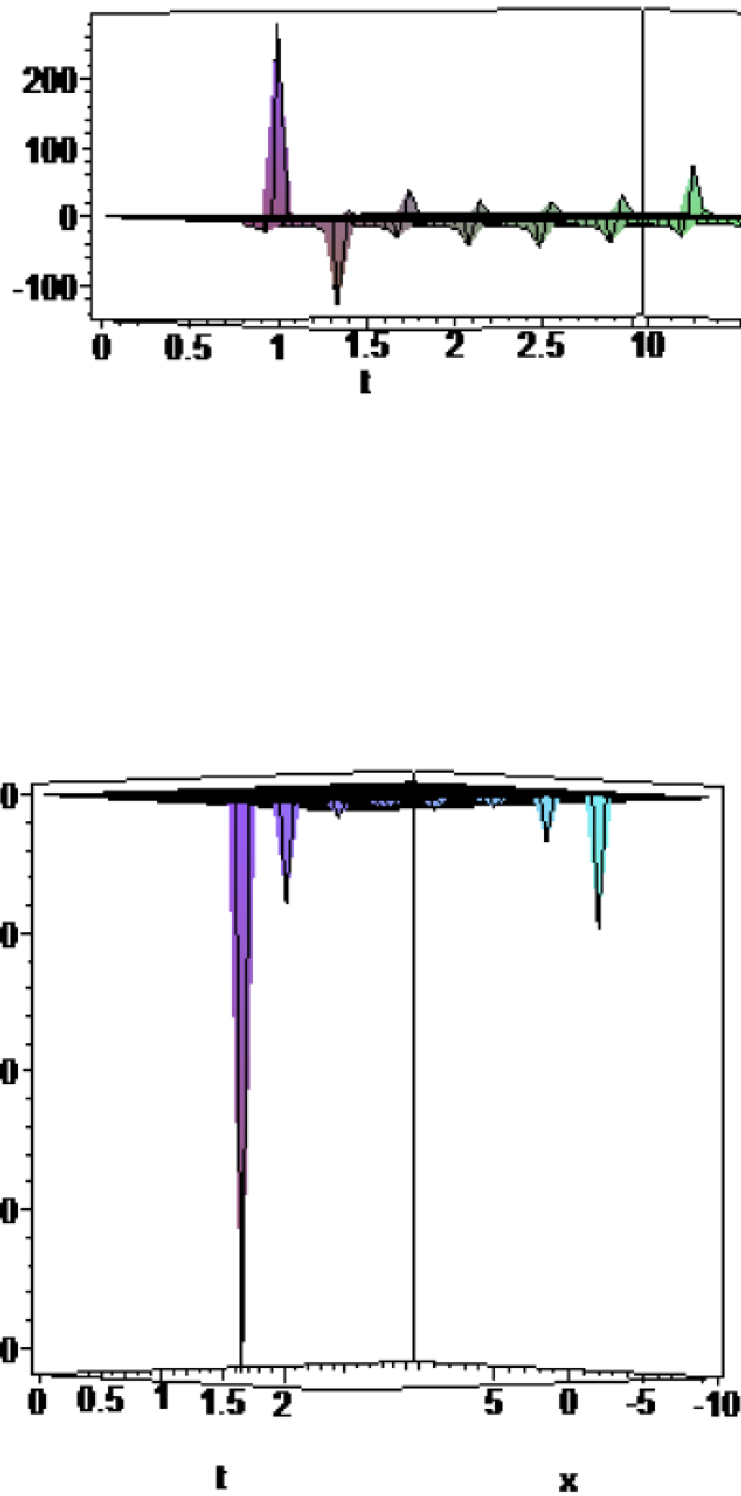

Figure 7

The figure of the solution for u(x; t) and v(x; t) obtained in Eq.(27) and Eq.(28) when C1= 4; b1= 1; α = 0:9:

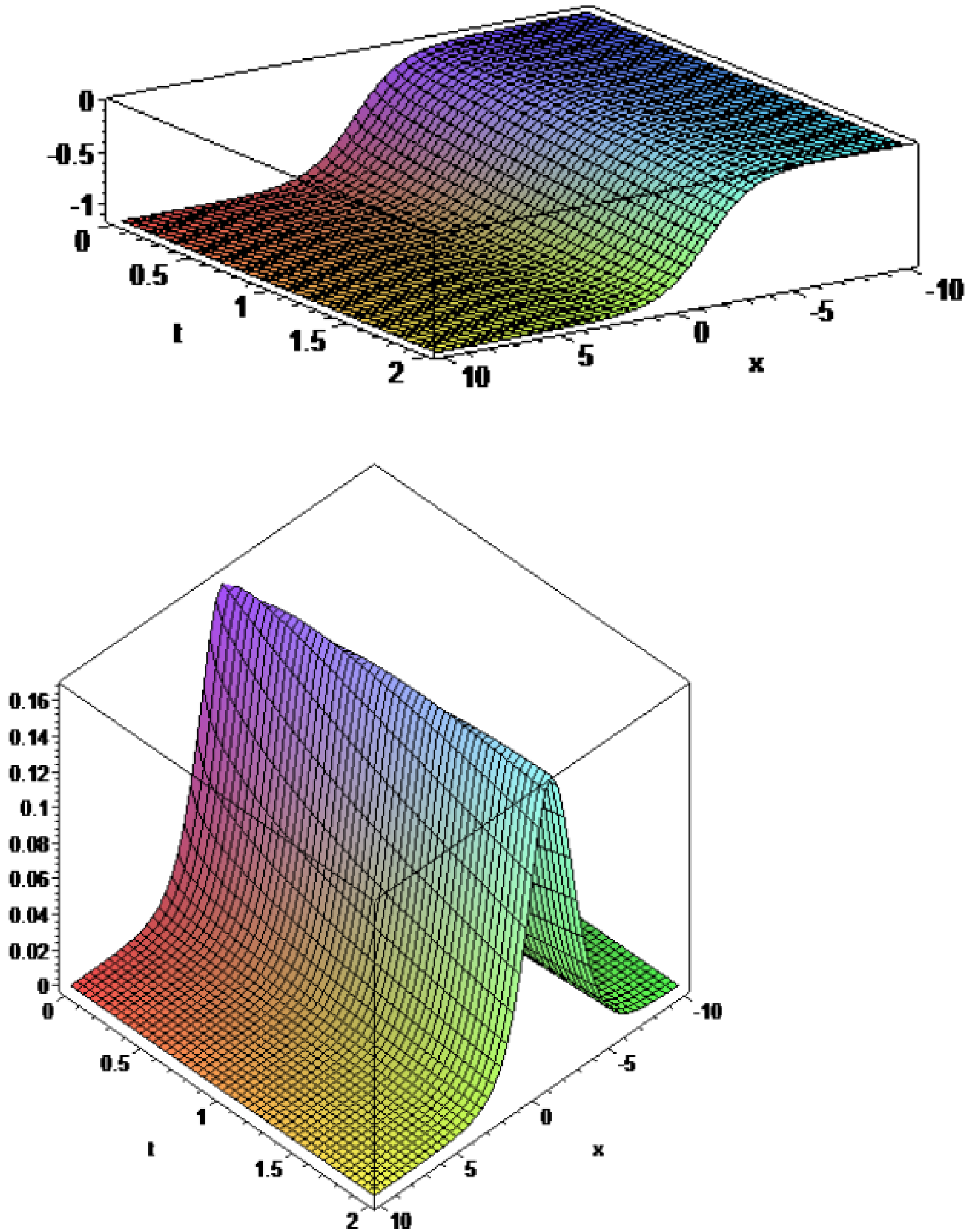

Figure 8

The figure of the solution for u(x; t) and v(x; t) obtained in Eq.(29) and Eq.(30) when C1= 1; b0= 1; b1= 1; α = 0:9:

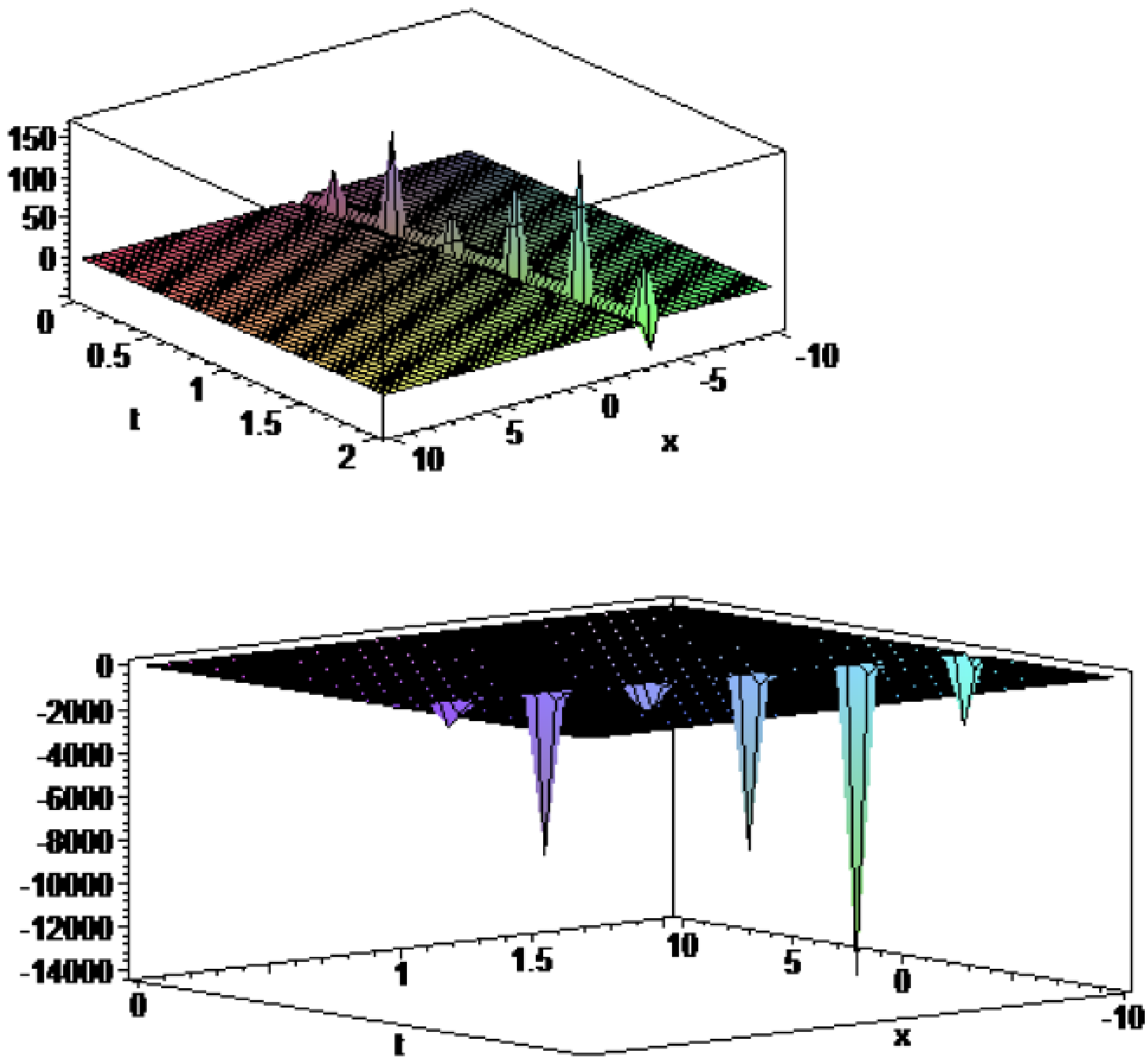

Figure 9

The figure of the solution for u(x; t) and v(x; t) obtained in Eq.(31) and Eq.(32) when C1= 4; b1= −1; α = 0:9: