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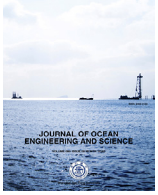

A novel investigation of exact solutions of the coupled nonlinear Schrodinger equations arising in ocean engineering, plasma waves, and nonlinear optics

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ABSTRACT

We have analyzed the two coupled nonlinear Schrödinger equations (CNLSE) in the current work. This model has applications in high birefringence fibers. To generate different types of analytical solutions, including exponential, periodic, and soliton-type, we operated generalized Kudryashov, modified Kudryashov and exponential rational function procedures. Thanks to this implementation, we contributed to ocean engineering, plasma waves, nonlinear birefringence phenomena, and pulse compression features. In addition, in our article, we have included 2D and 3D graphics of solutions to help understand the physical meaning of solutions. The article highlights the method's ability to provide various solutions to various physical problems.

1. Introduction

Complex phenomena that are encountered in economics, fluid dynamics, mathematical physics, optical fiber, and relativity have been modeled by using nonlinear partial differential equations (NPDEs) [1–3]. Generally, solving a NPDE is a difficult task. However, mathematicians have tried their best to make the task easy and promoted various techniques for its solutions [4–8]. For more studies in the sense of fractional calculus, the investigation of this topic with its applications is studied in [9–14]. They recommended many techniques, such as the Hirota bilinear technique [15], the improved F-expansion method [16], the $\exp(-\phi(\xi))$ technique [17], the auxiliary equation technique [18,19], the modified extended tanh technique [20], the improved generalized Riccati equation mapping technique [21], the sub-equation technique [22], the general projective Riccati equations technique [23], the first integral method [24], the sine-Gordon expansion approach [25], the transformed rational function technique [26], the symmetry reduction technique [27], and so on [28–31]. They have also graphed the solutions to support the physical behavior of each problem and the best of physical analysis. Riemann-Hilbert problems for nonlocal integrable equations derived by one group reduction [32–34] as well as for nonlocal integrable equations [35–37] derived from two group reductions, as an example, were used to analyze N-soliton solutions for nonlocal integrable equations.

In this regard, we presented and applied prominent and accurate analytical techniques, which are known as generalized Kudryashov, modified Kudryashov and exponential rational function procedures for the solution of CNLSEs. The manuscript highlights the method's ability to provide various solutions to various physical problems.

The two incoherent CNLSEs has many applications in ocean waves, and plasma physics. These equations are essential to the investigation of surface wave dynamic to understand some important

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oceanographic phenomena. The pulse propagation in a birefringent optical fiber is governed by the following:

$$\begin{aligned} &iq_{1t}+i\frac{\lambda}{2}q_{1x}+\frac{1}{2}q_{1xx}+(|q_1|^2+\mu|q_2|^2)q_1=0,\\ &iq_{2t}-i\frac{\lambda}{2}q_{2x}+\frac{1}{2}q_{2xx}+(|q_1|^2+\mu|q_2|^2)q_2=0, \end{aligned} \tag{1}$$

where the complex envelope amplitudes of the 2 modulated weakly resonant waves in 2 polarizations. are represented by the quantities q_1 and q_2. Here, quadratic dispersions and cubic terms represent the interactions. Due to the optical Kerr effect, the nonlinear terms of Eq. (1) describe the pulse compression and nonlinear birefringence. Various interesting phenomena, particularly nonlinear pulse stabilization, pedestal elimination in compressed pulses, and linear pulse splitting, are represented by CNLSEs. It has been presented that the two CNLSEs have application in birefringent fibers [38]. They have physical correlation in several scientific fields such as plasma physics, fluid mechanics, nonlinear optics, multi-component Bose-Einstein condensation, and optical fiber [39,40].

Some different solutions have been verified for this system. For example, Wazwaz has applied the variational iteration technique to Eq. (1) [48]. Wazwaz has founded optical envelope soliton solutions of this system [40].

While various phenomena in the field of oceanography have been studied in some recent research works concerning the perturbed nonlinear Schrödinger equation's complex soliton solutions in nonlinear optical fibers [41], the generalized Schrödinger-Boussinesq equations [42], the fuzzy fractional-order Boussinesq model [43], the fifth-order nonlinear wave equation [23], the Kaup-Boussinesq and generalized Hirota Satsuma coupled KdV systems [19], the $(2+1)$ dimensional Bogoyavlensky-Konopelchenko equation with variable-coefficient in wave propagation [44], the Mikhailov-Novikov-Wang equation [45], the fractional-order Liouville equation [16], nonlinear Schrödinger equation with higher dimension in the anomalous dispersion regime [46], and the cubic-quintic nonlinear Helmholtz equation [47], our research study is unique with novel results due to the fact that the investigated model is highly applicable in high birefringence fibers, and three employed methods: generalized Kudryashov, modified Kudryashov and exponential rational function procedures have been investigated in this work to obtain a variety of analytical solutions which are essential in understanding many scientific phenomena in ocean engineering, nonlinear birefringence phenomena, and pulse compression features.

The paper is constructed as: some preliminaries are mentioned in Section 2. Then, all our utilized techniques are described in Section 3. Our employed methods are applied and discussed in Section 4. Finally, the conclusion of the is drawn in Section 5.

2. Fundamental tools

some basic tools are introduced in this section to reduce a system of NPDE to an ordinary differential equation (ODE). Hence, the following system of NPDE is considered:

$$P^{\alpha}(q_i,(q_i)_t,(q_i),(q_i)_{tt},(q_i)_{xx},(q_i)_{xt},\ldots)=0, \tag{2}$$

where P^a inherits q_i and its partial derivatives, and q_i is a complex-valued function.

If we take $\alpha=2$ and $i=2$ in Eq. (2), the wave transformation is represented as:

$$\begin{aligned} q_1(x,t)&=u(\xi)e^{i(\alpha_1x-\beta_1t)},\\ q_2(x,t)&=v(\xi)e^{i(\alpha_2x-\beta_2t)},\\ \xi&=x-ct, \end{aligned} \tag{3}$$

where $\alpha_1,\alpha_2,\beta_1,\beta_2$ and c are constants to be determined. If we apply transformation (3) to Eq. (2) and both of the real and imaginary parts equated to 0, 2 equation systems are obtained. Then, the conditions for parameters are found by solving these 2 systems and using their parameters' values. Therefore, the following ODE is considered which will be later integrated w.r.t ξ possible times.

$$Q\left(u,u',u'',u''',\ldots\right)=0, \tag{4}$$

where the partial derivative is provided w.r.t ξ [49].

2.1. The generalized kudryashov procedure

The solution of Eq. (4) is assumed as:

$$u(\xi)=\frac{\sum\limits_{j=0}^{n}\rho_jQ^j(\xi)}{\sum\limits_{k=0}^{m}\sigma_kQ^k(\xi)},\ \rho_n\neq0,\sigma_m\neq0 \tag{5}$$

where $\rho_j(j=0,1,\ldots,n),\sigma_k(k=0,1,\ldots,m)$ are constants to be determined and the function $Q=Q(\xi)$ is the solution of

$$\frac{dQ}{d\xi}=Q^2(\xi)-Q(\xi), \tag{6}$$

which is

$$Q(\xi)=\frac{1}{1+Ae^{\xi}},A\text{ is integration constant.}$$

Here, we may find the positive integers n and m in Eq. (5) according to the homogeneous balance principle. Namely, we can balance the highest order derivatives and the highest power nonlinear terms in Eq. (10). Then, by substituting Eq. (5) into Eq. (4) with Eq. (6), we find a system of algebraic equations of $\rho_j(i=0,1,\ldots,n),\sigma_k(j=0,1,\ldots,m),\alpha_1,\alpha_2,\beta_1,\beta_2$ and c.Then, from the solutions of this system, we can extract new solutions of Eq. (4) [50].

2.2. The modified Kudryashov procedure

The solutions of Eq. (4) are assumed as:

$$u(\xi)=\sum_{j=0}^{m}\rho_j(\phi(\xi))^j,\ \rho_m\neq0, \tag{7}$$

where $\rho_j(j=0,1,\ldots,m)$ are constants to be determined, m is the balancing number, and the function $\phi(\xi)$ is:

$$\phi(\xi)=\frac{1}{1+\delta a^{\xi}}, \tag{8}$$

and Eq. (8) coincides with the 1st order ODE as follows:

$$\phi'(\xi)=\left(\phi^2(\xi)-\phi(\xi)\right)\ln a. \tag{9}$$

We may find the balancing number by the balance of the highest order derivatives and the highest power nonlinear terms in Eq. (4). If Eq. (7) is substituted into Eq. (8) along with Eq. (9), a set of algebraic equations is obtained for $\rho_j(j=0,1,2,\ldots,m),\delta,a,\alpha_1,\alpha_2,\beta_1,\beta_2$ and c. Then from the solutions of this system, various exact solutions of Eq. (4) are constructed [51,52].

2.3. The exponential rational function procedure

The solution of Eq. (4) is assumed as [53,54]:

$$u(\xi)=\sum_{j=0}^{m}\frac{\rho_j}{(1+e^{\xi})^j},\ \rho_m\neq0 \tag{10}$$

where $\rho_j (j = 0, 1, 2, \ldots, m)$ are constants will be determined and m is the balancing number as the previous procedure. If Eq. (10) is substituted into Eq. (4), a set of algebraic equations is obtained in parameters $\rho_j (j = 0, 1, 2, \ldots, m), \alpha_1, \alpha_2, \beta_1, \beta_2$ and c. Then, from the solutions of this system, we can extract new solutions of Eq. (4).

3. Applications

New traveling wave solutions of Eq. (1) are obtained in this section. Hence, we investigated the studied equation as follows:

3.1. Investigation

If Eq. (3) is substituted into Eq. (1), then the coefficients are collected together, and both of the imaginary and real parts are equated to 0, some relations are obtained as follows:

$$\begin{gathered}
\alpha_1 u' - cu' + \frac{\lambda}{2} u' = 0, \\
\left(-\frac{1}{2}\alpha_1^2 - \frac{1}{2}\alpha_1 \lambda + \beta_1\right) u + uv^2 \mu + u^3 + \frac{u''}{2} = 0, \\
\left(-c + \alpha_2 - \frac{\lambda}{2}\right) v' = 0, \\
-\left(\frac{1}{2}\alpha_2^2 - \alpha_2 \frac{\lambda}{2} - \beta_2\right) v + v^3 \mu + v^2 u + \frac{v''}{2} = 0.
\end{gathered} \tag{11}$$

We obtain α_1 and α_2 as follows from the first and third equations of solutions of this system:

$$\alpha_1 = c - \frac{\lambda}{2}, \alpha_2 = c + \frac{\lambda}{2}. \tag{12}$$

Then, from the remaining equations we find

$$u'' + 2u(u^2 + \mu v^2) + 2u(-\frac{1}{2}\alpha_1^2 - \frac{1}{2}\alpha_1 \lambda + \beta_1) = 0, \tag{13}$$

and

$$v'' + 2v(u^2 + \mu v^2) + 2v\left(-\frac{1}{2}\alpha_2^2 + \alpha_2 \frac{\lambda}{2} + \beta_2\right) = 0. \tag{14}$$

We can deduce from Eq. (13) and Eq. (14) that

$$u(\xi) = \pm v(\xi), \beta_1 = \beta_2. \tag{15}$$

If we substitute values (15) into (13), $m = 1$ is obtained.

3.2. The generalized Kudryashov procedure

In this subsection, we will use the generalized Kudryashov method to obtain the solutions of the Eq. (1). According to the method, the assumed solution is given by:

$$u(\xi) = \frac{\rho_0 + \rho_1 Q(\xi) + \rho_2 Q^2(\xi)}{\sigma_0 + \sigma_1 Q(\xi)}. \tag{16}$$

If we substitute Eq. (16) into Eq. (13) along with Eq. (6), then setting the coefficients of $Q^j(\xi)$ to zero, the following determining equation system is obtained.

$$Q^6(\xi) : 2\rho_2\sigma_1 + 2\rho_2^3(\mu + 1) = 0,$$

$$Q^5(\xi) : 6\rho_2\left(\sigma_0\sigma_1 - \frac{\sigma_1^2}{2} + 2\rho_1\rho_2(\mu + 1)\right) = 0,$$

$$Q^4(\xi) : 2\rho_2\left(\left(\frac{\lambda^2}{8} - \frac{c^2}{2} + \beta_1 + \frac{1}{2}\right)\sigma_1^2 - \frac{9\sigma_0\sigma_1}{2} + 3\rho_0\rho_2(\mu + 1) + 3\rho_1^2(\mu + 1) + 3\sigma_0^2\right) = 0,$$

$$Q^3(\xi) : \frac{\sigma_1^2\left(-4\rho_0 + \rho_1\left(-4c^2 + \lambda^2 + 8\beta_1\right)\right)}{4} + 4\sigma_0\sigma_1\left(\rho_2\left(\frac{\lambda^2}{8} - \frac{c^2}{2} + \beta_1 + \frac{3}{4}\right) - \frac{\rho_0}{2} + \frac{\rho_1}{4}\right) + 2\rho_1^3(\mu + 1) + \rho_1\left(2\sigma_0^2 + 12\rho_0\rho_2(\mu + 1)\right) - 10\rho_2\sigma_0^2 = 0,$$

$$Q^2(\xi) : \frac{\sigma_0^2\left(\rho_2\left(-4c^2 + \lambda^2 + 8\beta_1 + 16\right) - 12\rho_1\right)}{4} + 4\sigma_0\sigma_1\left(\frac{3\rho_0}{4} + \rho_1\left(\frac{\lambda^2}{8} - \frac{c^2}{2} + \beta_1 - \frac{1}{4}\right)\right) + 2\rho_0\left(3\rho_0\rho_2(\mu + 1) + \sigma_1^2\left(\frac{\lambda^2}{8} - \frac{c^2}{2} + \beta_1 + \frac{1}{2}\right) + 3\rho_1^2(\mu + 1)\right) = 0,$$

$$Q(\xi) : 2\rho_1\sigma_0^2\left(\frac{\lambda^2}{8} - \frac{c^2}{2} + \beta_1 + \frac{1}{2}\right) + 4\rho_0\sigma_0\sigma_1\left(\frac{\lambda^2}{8} - \frac{c^2}{2} + \beta_1 - \frac{1}{4}\right) + 6\rho_0^2\rho_1(\mu + 1) = 0,$$

$$Q^0(\xi) : 2\rho_0\left(\sigma_0^2\left(\frac{\lambda^2}{8} - \frac{c^2}{2} + \beta_1\right) + \rho_0^2(\mu + 1)\right) = 0.$$

If we solve the system, we get:

1)

$$\rho_0 = \pm\frac{\sigma_0}{(2\mu + 2)\sqrt{-\frac{1}{(\mu+1)}}}, \rho_1 = \mp\frac{2\sigma_0 - \sigma_1}{(2\mu + 2)\sqrt{-\frac{1}{(\mu+1)}}},$$

$$\rho_2 = \pm\sigma_1\sqrt{-\frac{1}{(\mu + 1)}}, \beta_1 = \frac{c^2}{2} - \frac{\lambda^2}{8} + \frac{1}{4}.$$

Therefore, the exact solutions of the system are given by:

$$(q_1)_{1,2} = \pm\frac{\left(Ae^{(x-ct)} - 1\right)e^{i\left(\left(c-\frac{\lambda}{2}\right)x-\left(-\frac{\lambda^2}{8}+\frac{c^2}{2}+\frac{1}{4}\right)t\right)}}{(2\mu + 2)\sqrt{-\frac{1}{(\mu+1)}}\left(Ae^{(x-ct)} + 1\right)}, \tag{17}$$

and

$$(q_2)_{1,2} = \pm\frac{\left(Ae^{(x-ct)} - 1\right)e^{i\left(\left(c+\frac{\lambda}{2}\right)x-\left(-\frac{\lambda^2}{8}+\frac{c^2}{2}+\frac{1}{4}\right)t\right)}}{(2\mu + 2)\sqrt{-\frac{1}{(\mu+1)}}\left(Ae^{(x-ct)} + 1\right)}. \tag{18}$$

2)

$$\rho_0 = 0, \rho_1 = \pm\frac{\sigma_1}{\sqrt{-\frac{1}{(\mu+1)}}(\mu + 1)},$$

$$\rho_2 = \pm\sigma_1\sqrt{-\frac{1}{(\mu + 1)}}, \beta_1 = \frac{c^2}{2} - \frac{\lambda^2}{8} - \frac{1}{2}.$$

Therefore, the exact solutions of the system are given by:

$$(q_1)_{3,4} = \mp\frac{2Ae^{(x-ct)}e^{i\left(\left(c-\frac{\lambda}{2}\right)x-\left(-\frac{\lambda^2}{8}+\frac{c^2}{2}-\frac{1}{2}\right)t\right)}}{(\mu + 1)\sqrt{-\frac{1}{(\mu+1)}}\left(A^2\left(e^{(x-ct)}\right)^2 - 1\right)}, \tag{19}$$

and

$$(q_2)_{3,4} = \mp\frac{2Ae^{(x-ct)}e^{i\left(\left(c+\frac{\lambda}{2}\right)x-\left(-\frac{\lambda^2}{8}+\frac{c^2}{2}-\frac{1}{2}\right)t\right)}}{(\mu + 1)\sqrt{-\frac{1}{(\mu+1)}}\left(A^2\left(e^{(x-ct)}\right)^2 - 1\right)}. \tag{20}$$

3)

$$\rho_0 = \pm\frac{\sigma_1\left(3\sqrt{-\frac{1}{(\mu+1)}}\sqrt{-\mu - 1} - 1\right)}{8\sqrt{-\mu - 1}},$$

$$\rho_1 = \pm\frac{\sigma_1}{\sqrt{-\frac{1}{(\mu+1)}}(\mu + 1)}, \rho_2 = \pm\sigma_1\sqrt{-\frac{1}{(\mu + 1)}},$$

$$\beta_1=\frac{4c^2\sqrt{-\mu-1}-\lambda^2\sqrt{-\mu-1}+3\sqrt{-\frac{1}{(\mu+1)}}\mu+3\sqrt{-\frac{1}{(\mu+1)}}+5\sqrt{-\mu-1}}{8\sqrt{-\mu-1}},\sigma_0=-\frac{\sigma_1}{2}.$$

Therefore, the exact solutions of the system are given by:

$$(q_1)_{5,6}=\mp\frac{\left(\left(3A^2e^{2(x-ct)}-2Ae^{(x-ct)}+3\right)\sqrt{-\mu-1}+\left(A^2e^{2(x-ct)}+2Ae^{(x-ct)}+1\right)(\mu+1)\sqrt{-\frac{1}{(\mu+1)}}\right)e^{i\left(\left(c-\frac{\lambda}{2}\right)x-\left(\frac{4c^2\sqrt{-\mu-1}-\lambda^2\sqrt{-\mu-1}+3\sqrt{-\frac{1}{(\mu+1)}}\mu+3\sqrt{-\frac{1}{(\mu+1)}}+5\sqrt{-\mu-1}}{8\sqrt{-\mu-1}}\right)t\right)}}{\sqrt{-\frac{1}{(\mu+1)}}(-\mu-1)^{3/2}\left(4A^2e^{2(x-ct)}-4\right)},$$

and

$$(q_2)_{5,6}=\mp\frac{\left(\left(3A^2e^{2(x-ct)}-2Ae^{(x-ct)}+3\right)\sqrt{-\mu-1}+\left(A^2e^{2(x-ct)}+2Ae^{(x-ct)}+1\right)(\mu+1)\sqrt{-\frac{1}{(\mu+1)}}\right)e^{i\left(\left(c+\frac{\lambda}{2}\right)x-\left(\frac{4c^2\sqrt{-\mu-1}-\lambda^2\sqrt{-\mu-1}+3\sqrt{-\frac{1}{(\mu+1)}}\mu+3\sqrt{-\frac{1}{(\mu+1)}}+5\sqrt{-\mu-1}}{8\sqrt{-\mu-1}}\right)t\right)}}{\sqrt{-\frac{1}{(\mu+1)}}(-\mu-1)^{3/2}\left(4A^2e^{2(x-ct)}-4\right)}.$$

4)

$$\rho_0=\pm\frac{\sigma_1\left(3\sqrt{-\frac{1}{(\mu+1)}}\sqrt{-\mu-1}+1\right)}{8\sqrt{-\mu-1}},$$

$$\rho_1=\pm\frac{\sigma_1}{\sqrt{-\frac{1}{(\mu+1)}}(\mu+1)},\ \rho_2=\pm\sigma_1\sqrt{-\frac{1}{(\mu+1)}},$$

$$\beta_1=\frac{4c^2\sqrt{-\mu-1}-\lambda^2\sqrt{-\mu-1}-3\sqrt{-\frac{1}{(\mu+1)}}\mu-3\sqrt{-\frac{1}{(\mu+1)}}+5\sqrt{-\mu-1}}{8\sqrt{-\mu-1}},$$

$$\sigma_0=-\frac{\sigma_1}{2}.$$

Therefore, the exact solutions of the system are given by:

$$(q_1)_{7,8}=\pm\frac{\left(\left(-3A^2e^{2(x-ct)}+2Ae^{(x-ct)}-3\right)\sqrt{-\mu-1}+(A^2e^{2(x-ct)}+2Ae^{(x-ct)}+1)(\mu+1)\sqrt{-\frac{1}{(\mu+1)}}\right)e^{i\left(\left(c-\frac{\lambda}{2}\right)x-\left(\frac{4c^2\sqrt{-\mu-1}-\lambda^2\sqrt{-\mu-1}-3\sqrt{-\frac{1}{(\mu+1)}}\mu-3\sqrt{-\frac{1}{(\mu+1)}}+5\sqrt{-\mu-1}}{8\sqrt{-\mu-1}}\right)t\right)}}{\sqrt{-\frac{1}{(\mu+1)}}(-\mu-1)^{3/2}\left(4A^2e^{2(x-ct)}-4\right)},$$

and

$$(q_2)_{7,8}=\frac{\left(\left(-3A^2e^{2(x-ct)}+2Ae^{(x-ct)}-3\right)\sqrt{-\mu-1}+\left(A^2e^{2(x-ct)}+2Ae^{(x-ct)}+1\right)(\mu+1)\sqrt{-\frac{1}{(\mu+1)}}\right)e^{i\left(\left(c+\frac{\lambda}{2}\right)x-\left(\frac{4c^2\sqrt{-\mu-1}-\lambda^2\sqrt{-\mu-1}-3\sqrt{-\frac{1}{(\mu+1)}}\mu-3\sqrt{-\frac{1}{(\mu+1)}}+5\sqrt{-\mu-1}}{8\sqrt{-\mu-1}}\right)t\right)}}{\sqrt{-\frac{1}{(\mu+1)}}(-\mu-1)^{3/2}\left(4A^2e^{2(x-ct)}-4\right)}.$$

Now, we will give the 3D an 2D graphs for solution $|(q_1)_{7,8}|$.

3.3. *The modified Kudryashov procedure*

In this subsection, we will use the modified Kudryashov method to obtain the solutions of the Eq. (1). According to the modified Kudryashov procedure, the assumed solution is represented as:

$$u(\xi)=\rho_0+\rho_1\phi(\xi). \tag{21}$$

Substituting Eq. (21) into Eq. (13) along with Eq. (9), then setting the coefficients of $\phi^j(\xi)$ to 0, the determining equation system is obtained as follows:

$$\phi^3(\xi):2\rho_1^3\mu+2\rho_1(\ln(a))^2+2\rho_1^3=0,$$

$$\phi^2(\xi):6\rho_0\mu\rho_1^2-3\rho_1(\ln(a))^2+6\rho_0\rho_1^2=0,$$

$$\phi(\xi):\rho_1(\ln(a))^2+6\rho_0^2\rho_1+6\mu\rho_0^2\rho_1-\rho_1c^2+\frac{\rho_1\lambda^2}{4}+2\beta_1\rho_1=0,$$

$$\phi^0(\xi):2\rho_0^3+2\mu\rho_0^3-\rho_0c^2+\frac{\rho_0\lambda^2}{4}+2\beta_1\rho_0=0.$$

If we solve the obtained system, we find:

$$\beta_1=-\frac{\lambda^2}{8}+\frac{c^2}{2}+\frac{(\ln(a))^2}{4},$$

$$\rho_0=\pm\sqrt{-\frac{1}{4+4\mu}}\ln(a),\ \rho_1=\mp2\sqrt{-\frac{1}{4+4\mu}}\ln(a). \tag{22}$$

The exact solutions of the considered system are given by:

$$(q_1)_{9,10}=\left(\pm\sqrt{-\frac{1}{4+4\mu}}\ln(a)\mp\frac{2\sqrt{-\frac{1}{4+4\mu}}\ln(a)}{1+\delta a^{(x-ct)}}\right)e^{i\left(\left(c-\frac{\lambda}{2}\right)x-\left(-\frac{\lambda^2}{8}+\frac{c^2}{2}+\frac{(\ln(a))^2}{4}\right)t\right)}, \tag{23}$$

and

$$(q_2)_{9,10}=\left(\pm\sqrt{-\frac{1}{4+4\mu}}\ln(a)\mp\frac{2\sqrt{-\frac{1}{4+4\mu}}\ln(a)}{1+\delta a^{(x-ct)}}\right)e^{i\left(\left(c+\frac{\lambda}{2}\right)x-\left(-\frac{\lambda^2}{8}+\frac{c^2}{2}+\frac{(\ln(a))^2}{4}\right)t\right)}. \tag{24}$$

Now, we will give graphical representation of $|(q_1)_{9,10}|$ in Fig. 2 as follows.

3.4. *The exponential rational function procedure*

In this subsection, we will apply the exponential rational function procedure to the Eq. (1). The desired solution for $u(\xi)$ takes the form:

$$u(\xi)=\rho_0+\frac{\rho_1}{1+e^{\xi}}. \tag{25}$$

From the substitution of Eq. (25) into Eq. (13) and collecting the all terms with the same order of the coefficients of exponential function together, we get the left-hand side of Eq. (4) another polynomial in exponential function. Then we equate each coefficient to 0 and find the system below:

$$e^{3\xi}:2\rho_0^3+2\rho_0\beta_1+2\mu\rho_0^3+\frac{\rho_0\lambda^2}{4}-\rho_0c^2=0,$$

$$\begin{aligned}e^{2\xi}:\ &\frac{3\rho_0\lambda^2}{4}+6\rho_0^2\rho_1+\frac{\rho_1\lambda^2}{4}+\rho_1+6\rho_0^3-3\rho_0c^2-\rho_1c^2\\&+6\rho_0\beta_1+6\mu\rho_0^2\rho_1+2\rho_1\beta_1+6\mu\rho_0^3=0,\end{aligned}$$

$$\begin{aligned}e^{\xi}:\ &12\mu\rho_0^2\rho_1+4\rho_1\beta_1+6\rho_0\mu\rho_1^2+6\rho_0^3+\frac{3\rho_0\lambda^2}{4}+6\rho_0\rho_1^2-\rho_1\\&+12\rho_0^2\rho_1-3\rho_0c^2+6\mu\rho_0^3+\frac{\rho_1\lambda^2}{2}-2\rho_1c^2+6\rho_0\beta_1=0,\end{aligned}$$

$$\begin{aligned}e^{0\xi}:\ &2\rho_1^3\mu-\rho_1c^2+\frac{\rho_1\lambda^2}{4}+2\rho_0^3+6\mu\rho_0^2\rho_1+6\rho_0\mu\rho_1^2-\rho_0c^2\\&+2\rho_1^3+2\mu\rho_0^3+\frac{\rho_0\lambda^2}{4}\\&+2\rho_0\beta_1+6\rho_0^2\rho_1+6\rho_0\rho_1^2+2\rho_1\beta_1=0.\end{aligned}$$

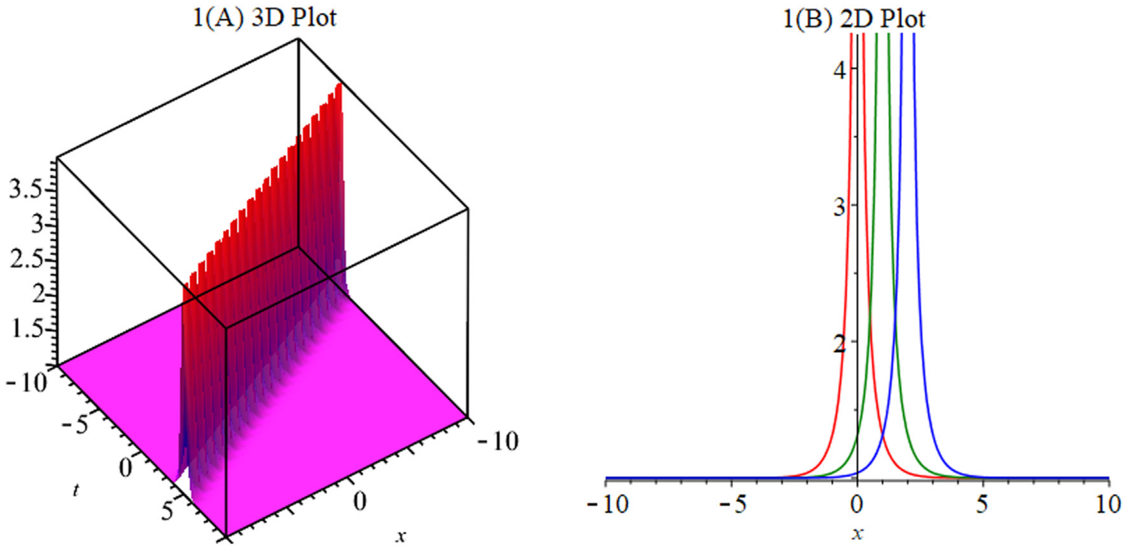

Fig. 1. Graphical representation of (a) $|(q_1)_{7,8}|$ for $\lambda = 3, \mu = -2, A = 1, c = 2$, and (b) red line was drawn when $t = 0$, green line was drawn when $t = 0.5$, and blue line was drawn when $t = 1$

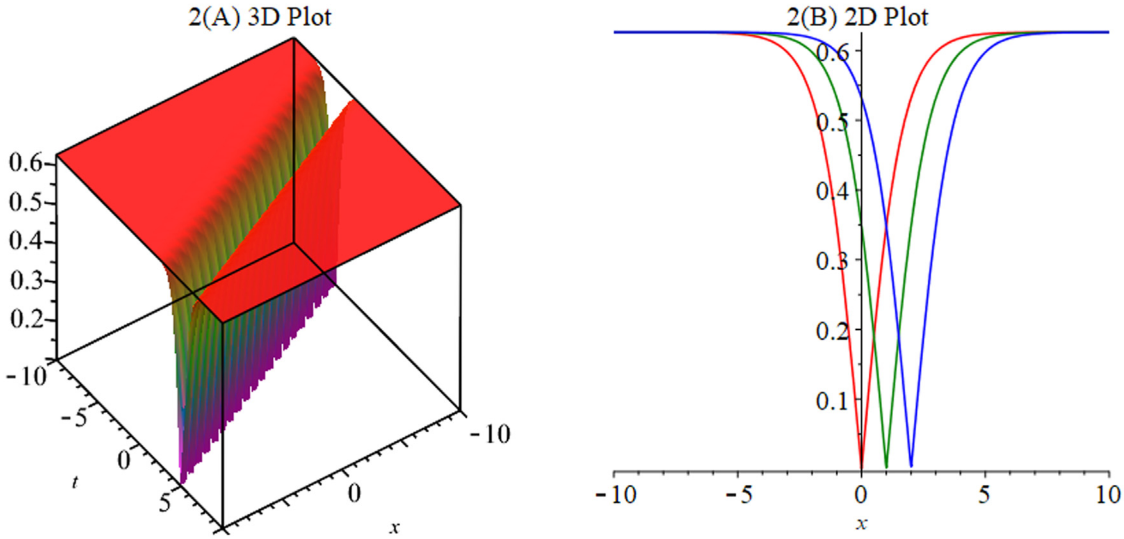

Fig. 2. Graphical representation of (a) $|(q_1)_{9,10}|$ for $\lambda = 3, \mu = -2, \delta = 1, c = 2, a = 3.5$, and (b) red line was drawn when $t = 0$, green line was drawn when $t = 0.5$, and blue line was drawn when $t = 1$

Thus, we obtain the following solutions:

$$\beta_1 = -\frac{\lambda^2}{8} + \frac{c^2}{2} + \frac{1}{4}, \rho_0 = \pm\sqrt{-\frac{1}{4+4\mu}}, \rho_1 = \mp 2\sqrt{-\frac{1}{4+4\mu}}. \tag{26}$$

The exact solutions of the considered system are given by:

$$(q_1)_{11,12} = \left(\pm\sqrt{-\frac{1}{4+4\mu}} \mp \frac{2\sqrt{-\frac{1}{4+4\mu}}}{1+e^{(x-ct)}} \right) e^{i\left(\left(c-\frac{\lambda}{2}\right)x - \left(-\frac{\lambda^2}{8}+\frac{c^2}{2}+\frac{1}{4}\right)t\right)}, \tag{27}$$

and

$$(q_2)_{11,12} = \left(\pm\sqrt{-\frac{1}{4+4\mu}} \mp \frac{2\sqrt{-\frac{1}{4+4\mu}}}{1+e^{(x-ct)}} \right) e^{i\left(\left(c+\frac{\lambda}{2}\right)x - \left(-\frac{\lambda^2}{8}+\frac{c^2}{2}+\frac{1}{4}\right)t\right)}. \tag{28}$$

Remark: If we check the obtained solutions, we may conclude that $(q_1)_{9,10}$ is generalization of the $(q_1)_{11,12}$.

4. Conclusion

In this work, we describe modified Kudryashov, generalized Kudryashov and exponential rational function procedures for generating a large number of innovative traveling wave solutions to CNLSE. We have also graph the solutions to support the physical behavior of the problem and the best of physical analysis. The resulting traveling wave solutions can be useful in theoretical investigations of the system under consideration, as they can explain various new wave properties. Thanks to this article, we have shown the applications of these three procedures and demonstrated their effectiveness in obtaining different types of solutions. In future studies, applications of other techniques to the equation discussed in this manuscript.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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