Journal of Mathematics Teacher Education (2026) 29:497–529
https://doi.org/10.1007/s10857-025-09697-z

Using general-mathematical and content-specific elements to assess the quality of prospective mathematics teachers' problems in a problem-posing context

Fadime Ulusoy[1]

Accepted: 14 April 2025 / Published online: 28 May 2025

Abstract

The open-ended nature of problem-posing tasks causes a serious diversity in learners' responses. This diversity presents a variety of difficulties in assessing the quality of the problems posed. This study aimed to assess the quality of problems that prospective mathematics teachers' (PMTs) posed about linear graphs through a combination of *general-mathematical* and *content-specific* elements. Additionally, this study concentrated on the PMTs' declarations about their difficulties in posing problems on linear graphs. In this study, the data sources were PMTs' problems, their own solutions to them, and reflection papers. The results revealed that integrating general-mathematical and content-specific elements in assessing the quality of problems provided a better understanding of PMTs' problem posing performances. In terms of general-mathematical elements, posed problems were mostly solvable, situated in a real-world context, and linguistically precise. However, the problems mostly failed to reflect content-specific elements related to key mathematical ideas about linear graphs. Moreover, the results showed that the mean score for general-mathematical elements was significantly higher than the mean score for content-specific elements in the problems. In their written explanations, PMTs also said that task-related components (like prompts and situations) and teacher-related components (like lack of experience, weakness in subject knowledge, and pedagogical content knowledge) made it challenging for them to pose problems about linear graphs. It was concluded that the inclusion of both general-mathematical and content-specific elements in evaluating the quality of the posed problems paves the way for future empirical and theoretical studies that seek to develop a systematic approach to problem assessment.

Keywords Problem posing · Prospective mathematics teachers · Assessment · Problem quality · General-mathematical elements · And content-specific elements

✉ Fadime Ulusoy
fadimebayik@gmail.com; fadime@kastamonu.edu.tr

[1] Department of Mathematics and Science Education, Faculty of Education, Kastamonu University, 37200 Kastamonu, Turkey

Introduction

The inclusion of problem posing in the school mathematics curriculum and pedagogy can be seen as a relatively new endeavor compared to the integration of problem solving (Brown & Walter, 2005; Silver, 1994). Nonetheless, educators, researchers, and curriculum frameworks have regarded problem posing as a vital cognitive endeavor integral to high-quality mathematics, and they have increasingly focused on it in the learning and teaching of mathematics over the last two decades (Cai et al., 2015; Zhang et al., 2025). Previous research addresses several arguments for the significance of problem posing for all mathematics learners and prospective/in-service teachers. For example, problem posing can enhance students' conceptual understanding of mathematical problem-solving skills and creativity (Bonotto & Santo, 2015; Van Harpen & Sriraman, 2013). Similarly, there are many approaches using problem posing in teacher education, such as a goal of instruction or a goal to build mathematical or pedagogical content knowledge (Kilic, 2015; Lee et al., 2018).

Among the numerous reasons presented by researchers, problem posing is also proposed as a method to assess learners' understanding of mathematics (Cai & Hwang, 2020; Silver, 1994). In this sense, Kwek (2015) posited that problem posing could be used as "a formative assessment tool to examine students' thinking processes, understandings, and competencies" (p. 273). However, the open-ended nature of problem-posing tasks causes a serious diversity in learners' answers (English & Halford, 1995; Lin, 2004). Although this diversity is preferable for enriching teaching, it also brings various difficulties in assessing the quality of problems. To understand the quality of the problems posed, previous studies have mostly examined the problems according to *general-mathematical elements* that are not specifically associated with a mathematical topic or subject. In other words, general-mathematical elements can be applied to a range of mathematical topics or learning areas (e.g., algebra, geometry) while assessing the problems posed by them. The label "general-mathematical" represents the elements that can potentially be used in a range of mathematical topics while assessing the quality of the posed problems. Some of these elements are *solvability* (e.g., English, 1997), *plausibility* (e.g., English, 1998; Leung & Silver, 1997), and *language use* (e.g., Silver & Cai, 1996). These studies revealed that students at all grade levels as well as the majority of teachers are competent at posing mathematical problems based on these general-mathematical elements (Cai et al., 2015; Silver & Cai, 1996).

Some posed problems may be mathematical, solvable, make sense, and use correct language, but they may not relate to the mathematical concept in the problem-posing task or may not be mathematically appropriate for the situation (e.g., posing a multiplication problem instead of a fraction addition problem; see Calabrese et al., 2022). In such cases, it is necessary to consider specific elements based on the mathematical concept of the task in addition to general-mathematical elements to assess the quality of the problems. In recent years, researchers have begun to assess the quality of the problems by examining *content-specific elements* associated with a specific mathematical topic, particularly when posing problems related to specific subjects. In other words, they evaluate the problems based on how they present key mathematical concepts, taking into account the mathematical content of the problem-posing task, such as linear graphs and multiplication (e.g., Cai et al., 2013; Calabrese et al., 2022; Silber & Cai, 2021).

Teachers are responsible for posing mathematically worthwhile problems in the teaching of mathematics since "effective problem posing is critical to high-quality mathematics teaching" (Cai & Hwang, 2020, p. 1). By posing problems, teachers can shape students' cognitive processes in their classrooms (Cai et al., 2015; Crespo, 2003; Crespo & Harper, 2020), promote creative thinking (Bonotto & Santo, 2015; Leung & Silver, 1997), learn new-for-them mathematics (Koichu, 2020), and produce valuable classroom resources (Marco & Palatnik, 2024). Despite its potential, posing problems is a challenge for both prospective and in-service teachers, particularly in the absence of specific numerical information and in the presence of a requirement for real-life context in problem-posing tasks (e.g., Leung & Silver, 1997; Silber & Cai, 2021; Stickles, 2011; Ulusoy, 2023). Before expecting teachers to independently pose worthwhile problems, it might be useful to understand the quality of their posed problems. However, there exists no standardized measure or method for assessing the quality of posed problems (Marco & Palatnik, 2024; Zhang et al., 2025). Yet, the identification of quality elements for the assessment of teachers' problem-posing products is crucial for the development of teachers' problem-posing proficiency and for the design of effective professional development implementations situated within initial teacher education programs where they can experience analyzing problems posed by students from the point of view of various quality elements (Leavy & Hourigan, 2022a; Marco & Palatnik, 2024). The current study aimed to assess the quality of problems that prospective mathematics teachers (PMTs) posed about linear graphs through a combination of *general-mathematical* and *content-specific elements*. Therefore, the results of this study can be instructive for empirical and theoretical future studies that aim to find a systematic way to assess the quality of posed problems.

The present study makes valuable contributions to the existing body of research on problem posing in several ways. Linear graphs have become more common in scientific textbooks, research publications, official scientific communication venues, and national and international examinations such as PISA and TIMSS (Lowrie & Diezmann, 2009; Peden & Hausmann, 2000). Previous scholarly investigations have observed that students from middle school to college face diverse challenges in generating and comprehending linear graphs. These challenges include perceiving a graph as a literal representation, conflating the concepts of slope and height, and mistaking the distinction between an interval and a point (Glazer, 2011; Kerslake, 1977; Leinhardt et al., 1990). Herein, teachers have a responsibility to provide opportunities for students to expand their knowledge (Bayazit & Gray, 2006). For this reason, they are expected to have a robust understanding of linear graphs to support their students' knowledge (Patahuddin & Lowrie, 2019). Analyzing the problems posed by PMTs in terms of content-specific elements will provide important information about the extent to which they can reflect key mathematical elements related to linear graphs in their problems. This may also be useful in making inferences about their content knowledge about linear graphs.

The current study extends the previous research because general-mathematical and content-specific elements were used together to assess the quality of the problems that PMTs posed. The potential and efficacy of integrating general-mathematical and content-specific elements when assessing the quality of problems posed appear to be promising and fruitful for a multitude of reasons. Problem posing is a complex process; hence, researchers need clear, understandable, and applicable criteria to assess the problems posed. In particular, for teachers not accustomed to using open-ended activities, understanding the mathematical, contextual, and linguistic features of the posed problems can turn into an unknown and complex action (Kwek, 2015; Lin, 2004). Hence, the hybrid use of these elements might provide opportunities for researchers and teachers to enhance their vision about how to effectively assess the quality of problems. Therefore, assessing the quality of the problems posed through both general-mathematical and content-specific elements can have a more functional role in understanding learners' thinking, errors, misconceptions, and/or deficiencies regarding the mathematical concept in a problem-posing activity. In particular, the following questions were developed in accordance with the study's goal and significance: (1) What is the quality of problems that PMTs posed related to linear graphs in terms of general-mathematical elements (solvability, context, and language)? (2) What is the quality of problems that PMTs posed related to linear graphs in terms of content-specific elements (covariation, linearity, and y-intercepts)? (3) What is the difference between scores on general-mathematical and content-specific elements in the posed problems? (4) What do PMTs declare as difficulties they encounter while posing problems related to linear graphs?.

Theoretical background

Problem posing

Researchers have proposed numerous notable definitions for mathematical problem posing (Cai & Hwang, 2020; Fuson, 1992; Silver, 1994; Stoyanova & Ellerton, 1996). For instance, some researchers define problem posing as creating a new problem to discover a given situation or reformulating an existing problem, which can occur before, during, or after a problem-solving process (Silver, 1994). This is similar to what Cai and Hwang (2020) said about problem posing: "several related types of activity that entail or support teachers and students formulating (or reformulating) and expressing a problem or task based on a particular context" (p. 2). According to Stoyanova and Ellerton (1996), problem posing is a "process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems" (p. 518). Fuson (1992), on the other hand, said that posing a math problem means "translating from the natural language representation of a problem to the mathematical language representation of the model" (p. 285). In this study, Fuson's definition was adopted as an underlying understanding because the task particularly emphasizes the interaction between the graphical representation of a problem and the natural and mathematical language representation.

According to Stoyanova and Ellerton (1996), there are three categories of problem-posing situations: *free*, *semi-structured*, and *structured*. In a structured situation, educators ask learners to formulate a problem that aligns with a specific problem. A semi-structured situation presents learners with an equation, a picture, a graph, a table, or a calculation. In such situations, they are requested to examine the structure of the given situation and complete it using their knowledge, skills, and prior experiences in mathematics (Van Harpen & Sriraman, 2013). Besides, learners are requested to pose a problem without restrictions or over a general situation in a free situation. In this study, as a semi-structured problem-posing task, a qualitative interpretation task was used to examine PMTs' conceptual understanding of the key mathematical elements related to linear graphs in the problem-posing context due to the following reasons. Instead of looking at exact numbers, these kinds of graph tasks require looking at the whole graph to understand the pattern of covariation between the two variables (Cai et al., 2013; Leinhardt et al., 1990). Such tasks provide opportunities to assess learners' conceptual understanding about the functional relationship between variables beyond the simple use of letters x and y as placeholders (Cai et al., 2013; Kar, 2016; Silber & Cai, 2021). In addition, qualitative graph tasks show how well students understand what the y-intercept and slope mean in real life.

Assessing the quality of problems in the problem-posing context

Numerous frameworks have been proposed to assess the problems posed by learners (Cankoy & Özder, 2017; English, 1997; Leung & Silver, 1997; Silver & Cai, 1996). In these frameworks, when analyzing the quality of problems, researchers have mostly focused on the elements of *solvability* (English, 1997; Leung & Silver, 1997; Silver & Cai, 1996). However, although a problem can be solved using the given information, the answer may be unreasonable and possess a structure that is not applicable in real-world scenarios. Therefore, numerous scholars argue that it is essential to assess the *reasonability* of a given situation (Cai et al., 2013; English, 1998; Silver, 1994), *mathematical structure* (Koedinger & Nathan, 2004; Riley & Greeno, 1988), and *mathematical complexity (cognitive complexity)* (Kwek, 2015). Moreover, the literature suggests that *contextual factors* (Singer et al., 2013; Van Harpen & Presmeg, 2013) and the *language use* in posed problems (English, 1998; Van Harpen & Presmeg, 2013) are important considerations in their assessment of the quality. For example, Silver and Cai (1996) discussed whether the posed problems are primarily mathematical problems (see Fig. 1a). Then, they examined the problems in terms of *solvability* and *language use*. Apart from this, Leung and Silver (1997) also considered mathematical problems in terms of *plausibility* (Fig. 1b). In addition, they examined *the number of operations (operational complexity)* required to solve the problem.

Cankoy and Özder (2017) developed a rubric to understand the quality of students' posed problems. The framework has five dimensions: *solvability*, *reasonability*, *mathematical structure*, *context*, and *language*. Each dimension also has two subcategories (see Table 1). In this theoretical framework, each dimension is handled separately and scored as 1 and 0 when assessing the quality of the problems.

Leavy and Hourigan (2022a) recently suggested a framework for formulating elementary mathematics problems, outlining the desirable characteristics of such problems. The framework has eight indicators, each signifying a characteristic of high-quality mathematics problems. These include the use of motivating and engaging context, clarity in language and cultural context, curriculum coherence, attention to cognitive demand, etc. Certain indicators are closely associated with the characteristics examined in the previous studies. Several studies have explored the quality of the problems learners pose based on the

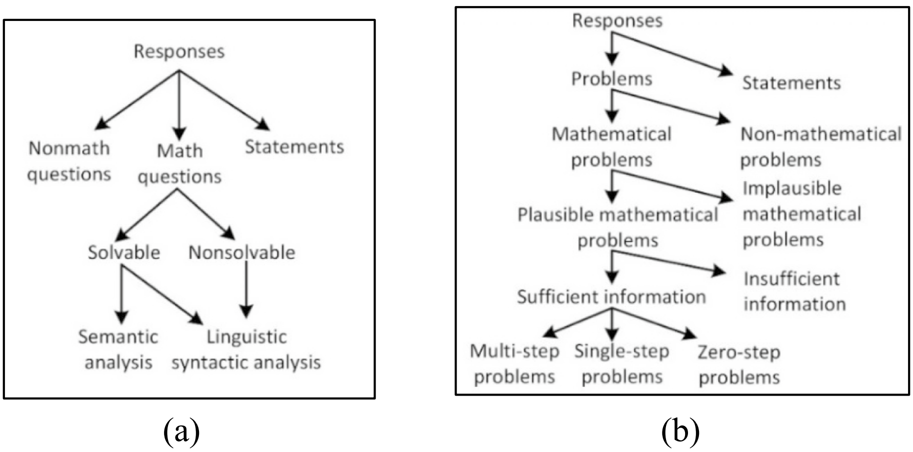


Fig. 1 Initial frameworks for assessment of the problems posed

Table 1 Scoring rubric for problem-posing skills (Cankoy & Özder, 2017)

Dimension	Sub-category	Explanation
Solvability	Solvable	The given information in the posed problem is adequate to solve it
	Unsolvable	The given information in the posed problem is not adequate to solve it
Reasonability	Reasonable	The posed problem and its solution are reasonable and applicable in the real world
	Unreasonable	The posed problem and its solution are not reasonable and applicable in the real world
Mathematical structure	Result unknown model	The unknown variable of the posed problem is at the end of the problem
	Start unknown model	The unknown variable of the posed problem is at the beginning of the problem
Context	Routine	The situation in which the posed problem was located was frequently used by the teacher in the classroom, or frequently seen in the textbook
	Non-routine	The situation in which the posed problem was located was rarely used by the teacher in the classroom or hardly seen in the textbook
Language	Clarity-Understandability	The posed problem is clear and understandable
		The posed problem is not clear and not understandable
	Obeying grammar rules	The language of the posed problem is grammatically correct. The posed problem has some grammatical mistakes
		The language of the posed problem is grammatically correct. The posed problem has some grammatical mistakes

aforementioned frameworks. According to the literature, it was observed that a majority of the problems generated by teachers or prospective teachers were solvable (e.g., Cai et al., 2020; Lee, 2021; Li et al., 2020; Wessman-Enzinger & Tobias, 2022), regardless of the mathematical concept in the problem-posing context. Moreover, studies have revealed that the vast majority of problems posed by students were clear (e.g., Nedaei et al., 2022; Silver & Cai, 1996) and situated in real-life context (e.g., Silber & Cai, 2021). Overall, although students can offer solvable questions, these are frequently straightforward and have low mathematical complexity (Kwek, 2015).

In recent years, researchers have begun to focus on *content-specific elements* (key mathematical ideas) in addition to *general-mathematical elements* in posed problems to assess the quality of the problems or learners' conceptual understanding about mathematical topics (e.g., linear graphs (Kar, 2016; Silber & Cai, 2021); multiplication (Calabrese et al., 2022); algebraic symbolism (Cañadas et al., 2018); integer addition and subtraction (Wessman-Enzinger & Tobias, 2022)). For example, Cañadas et al. (2018) examined how prospective teachers give meaning to algebraic symbolism and discovered that a majority of them generated problems with syntactic structures that differed from the ones provided. Wessman-Enzinger and Tobias (2022) also reported that teachers encountered difficulties in conceptualizing several contextual problem types that accurately and realistically corresponded to distinct integer operations. These studies showed that it is necessary to examine key mathematical elements in posed problems to provide detailed analysis about the quality of PMTs' posed problems. Moreover, analyzing teachers' mathematical problems from the point of view of the desirable features embedded in the problems is crucial in terms of developing their problem-posing proficiency and knowledge for teaching in effective professional development experiences (Leavy & Hourigan, 2022a).

Key mathematical ideas related to linear graphs

Comprehending linear graphs is critical for students developing a solid foundation for the content of mathematics areas such as algebra and geometry (National Council of Teachers of Mathematics [NCTM], 2000). To understand linear graphs, students need to know about important mathematical ideas like how variables x and y change together, parameters m (which stands for "rate of change") and c (which stands for "y-intercept") in both math and real life (Pierce et al., 2010; Silber & Cai, 2021). Pierce et al. (2010) describe the main mathematical parts of the graphs of linear functions in the standard equation $y = mx + c$. These are (i) two related variables, x and y, both of degree 1, where the value of y depends on the value of (ii) c, which is a constant and the y-intercept for a function graph; and (iii) m, which is the rate of change and the gradient for a function graph. These elements can be considered from their symbolic, numerical, graphical, and real-world perspectives (see Table 2).

When presented in real-life contexts (Billings & Klanderman, 2000; Glazer, 2011), linear graphs can be more challenging to understand than other graph representations (Ali & Peebles, 2013; Peebles & Ali, 2015). For example, it is necessary to coordinate the values of two variables simultaneously for each point when reading a linear graph (Leinhardt et al., 1990). Moreover, in a linear graph, it is necessary to interpret between and beyond visible data points due to the covariation between variables (Boote & Boote, 2017; Leinhardt et al., 1990). In real-world contexts, m may always be understood as a rate of change. On the other hand, c may represent a starting or initial value.

Table 2 Four dimensions of m and c (Bardini & Stacey, 2006, p. 115)

	Symbolic	Graphical	Numerical	Context
m	The equation $y = mx + c$. It can either be a letter or a number	The gradient of the graph of $y = mx + c$. The slope of a line	The ratio $\Delta y/\Delta x$	The meaning of m in a real-world context (e.g., a speed, hourly rate of pay)
c	The constant term of the equation. It can either be a letter or a number	The y-intercept. It is a coordinate of a point on the line	The value of y when $x = 0$	The meaning of c in a real-world context (e.g., taxi ‘flag fall’, 32° F in temperature conversion)

Teachers' understanding of linear graphs

Previous studies have pointed out that students' incomplete understanding of linear functions and graphs is numerous and diverse (Davis, 2007; Hattikudur et al., 2012; Leinhardt et al., 1990; Pierce et al., 2010; Stump, 2001). Many students have trouble finding a covarying relationship between variables (Wavering, 1989), and understanding what a continuous graph means because they are used to reading graphs point by point (Bell & Janvier, 1981; Leinhardt et al., 1990). Yet, research about teachers' understanding of linear graphs is much less developed than similar research on student interpretations. For example, Patahuddin and Lowrie (2019) examined sixty-one Indonesian middle school teachers' comprehension of interpreting a context-based line graph, based on their gender and years of teaching experience. They found no differences in teachers' understanding based on gender or years of teaching experience. They also found that most of the teachers interpreted the graph as an iconic representation of a real-life condition instead of an abstract presentation of data (i.e., speed vs. time). Patahuddin and Lowrie (2019) also revealed that teachers interpreted the intersection of two lines as the two cars meeting at one place in a speed-time graph.

Previous research also found that some prospective teachers see a graph as a picture rather than abstract quantitative information (Glazer, 2011), get slope and height mixed up (Ritter & Coleman, 1995), can't figure out what the variables are (Bowen & Roth, 2005), and the graph doesn't have linearity or y-intercepts in the right place (Ritter & Coleman, 1995). However, teachers must comprehend a linear graph as a representation of the connection between two variables that change simultaneously. They should be capable of articulating this relationship not only via numerical values or equations but also, more significantly, through verbal descriptions. Thus, it is a valid argument that teachers who struggle with interpreting line graphs may be unable to assist their students in overcoming comparable challenges. For this reason, researchers suggest that professional development programs should include opportunities for both pre-service and experienced teachers to actively participate in context-based graph exercises (Patahuddin & Lowrie, 2019; Silber & Cai, 2021; Ulusoy, 2023). The present study not only provides valuable insight into what PMTs think about linear graphs but also offers a novel approach to assessing problems posed by combining general-mathematical and content-specific elements. In summary, the related literature has tried to explain teachers' understanding of linear graphs by focusing on the difficulties or deficiencies they experience. However, knowing or understanding a mathematical concept does not necessarily mean being able to reflect the key mathematical elements related to it in a problem-posing context. This study is different from others because it looks at how well PMTs can use key mathematical concepts related to linear graphs in their problems instead of finding out what they know or don't know.

Teachers' problem-posing performance about linear graphs

In the related literature, there are a few studies on linear graphs in the context of problem posing (Cai et al., 2013; Huang & Kulm, 2012; Işık et al., 2011, 2012; Kar, 2016; Silber & Cai, 2021; Ulusoy, 2023). Those studies indicate that the problem posed by students and PMTs failed in terms of expressing key mathematical ideas like y-intercepts and slopes in their posed problems. For instance, Cai et al. (2013) depicted a linear graph in a purely mathematical context but requested USA secondary school students to provide real-life situation that could be represented by the graph. Within the given task, it is evident that certain fundamental mathematical concepts pertaining to the line can be identified, despite the absence of labeled axes. These concepts include the presence of a non-zero y-intercept, which signifies a starting point on the line, as well as the indication of a positive relationship, denoting an upward trend. Additionally, the slope of the line can be interpreted as representing a proportion of 1:2, further contributing to the understanding of its mathematical properties. They found that most of the problems posed do not accurately reflect the starting value and rate of change in the graphs. The researchers discovered that a mere 19% of the students' problems included the concept of linear relationships correctly. Further, although the slope value on the graphical paper was presented in an easily calculable way, the linear relationship was reflected in only 26.9% of the students' problems.

Kar (2016) examined 93 Turkish prospective elementary mathematics teachers' problems while dealing with linear graphs in real-world scenarios. In posed problems, the researcher found some shortcomings as follows: (a) failing to reflect the linear relationship, (b) omitting the initial points of the lines, (c) contradictions between the problem narrative and the given graph, and (d) presenting a problem without a fundamental question. Moreover, Işık et al. (2012) asked 80 Turkish primary prospective teachers to pose real-life problems about three line graphs that vary in terms of starting points, breaking points, and slope values. They found that 53% of the problems posed could not reflect linearity due to the use of categorical variables. Many of the problems that were given did not have the right y-intercept and x-intercept, did not state the breaking points, and did not have a question root when they were given in a real-life setting (Işık et al., 2012; Kar, 2016). Silber and Cai (2021) also examined mathematical ideas that USA undergraduate students invoked when posing problems about a graph task not contextualized in a real-life situation. They asked them to add context to the graph while posing problems to analyze which pieces of mathematical information they could identify and interpret in context. Based on the task, they examined the following key mathematical ideas: the y-intercept, the increasing nature of the relationship, and (iii) the slope. Their study revealed that reflecting the covariation between two variables was relatively easier than reflecting the slope in posed problems.

In terms of the task, Kar (2016) asserted that the quality of posed problems declined as the graph's complexity increased. Put simply, posing real-life problems in a linear graph task, including two linear relationships with different starting points, was found more challenging in terms of reflecting key mathematical elements compared to a graph that just has one line. Researchers propose that future studies should investigate the nature of learners' posed problems related to two linear relationships depicted in a single graph (Cai et al., 2013; Kar, 2016). Consistent with this proposal, the present work investigates issues within the context of a semi-structured problem-posing scenario, where a single graph represents two distinct linear relationships.

Methods

This study employed an exploratory research design (Stebbins, 2001) to examine the quality of problems posed by prospective mathematics teachers (PMTs) based on general-mathematical and content-specific elements. An exploratory study is "a broad-ranging, purposive, systematic prearranged undertaking designed to maximize the discovery of generalizations leading to description and understanding" (Stebbins, 2001, p. 3). Exploratory research design was considered suitable for this study as it allows for an in-depth analysis of an emerging phenomenon characterized by a lack of detailed preliminary research within the research topic. In this regard, this study was evaluated as an attempt to lay the groundwork that will lead to developing extensive studies on assessing problem-posing skills through the general-mathematical and content-specific elements.

Context and participants

The participants consisted of 47 prospective fourth-grade middle school mathematics teachers in a teacher preparation program at a state university in northern Turkey. They aged between 22 and 24, with an average of 22.3 years. They participated in the study voluntarily. They had completed all common educational courses and most program-specific courses. Figure 2 presents the undergraduate courses they took, which included linear functions.

In the Turkish mathematics curriculum (Ministry of National Education [MoNE], 2018), middle scholars (grades 5–8) encounter proportional and non-proportional situations in grade 7. They also examine covariation in numerical and figural patterns at the seventh-grade level. In eighth grade, students must learn how to show linear functions in different ways and understand basic mathematical ideas related to them (like the slope) in both a mathematical and a real-world setting. Some objectives at the eighth-grade level are as follows: students are able to "express how one variable changes depending on the other in a linear relationship with a table and an equation", "create and interpret equations, tables, and graphs of real-life situations with linear relationships," and "construct the graphs of linear functions" (p. 73). To conclude, middle school teachers are expected to know major mathematical ideas related to linear functions and graphs both mathematically and pedagogically.

Problem-posing task

Problem-posing tasks consist of two parts: *problem situations* and *prompts*, where the problem situation provides the context and data used to pose the problem and the prompt indicates what is expected when problem posing (Cai et al., 2023; Leavy & Hourigan, 2024). In this study, the problem-posing task was adapted from Cai et al.'s (2013) study. Some studies emphasize that proportional situations in graphs become simple for learners (Cai et al., 2013; van Dooren et al., 2005). For example, Cai et al. (2013) suggest that it is necessary to investigate learners' competencies in a problem-posing situation where two linear relationships are

The courses including linear functions taken the undergraduate program		
Pure mathematics courses	***Applied mathematics courses***	***Teaching mathematics courses***
• Fundamentals of Mathematics • Calculus I-II • Analytic Geometry I-II	• Basic Physics I-II	• Methods of Teaching Mathematics in Middle Schools I-II • School Experience
Learning and teaching opportunities with regard to linear functions		
Linear functions are mostly studied in the context of abstract mathematics rather than identifying, formulating and solving problems in scientific contexts (e.g., they learn slope as a geometric ratio, geometric property, algebraic formula, a functional property, a parametric coefficient, trigonometric and calculus conception).	Linear functions are mostly studied in kinematics-based contexts (e.g., position, velocity, acceleration) rather than in abstract math contexts.	Linear functions are experienced pedagogically (e.g., students' learning difficulties, instructional ways, lesson planning) by observing and reflecting on instructional activities in a classroom environment at middle schools

Fig. 2 Participants' educational background in terms of linear functions

given on a single graph. For this reason, in the task, the problem *situation* was less structured and included two linear relationships in a graph without numerical data. Despite the lack of labeled axes, it is possible to discern key mathematical ideas about the lines. In the task, (i) while the y-intercept of a line is zero, the y-intercept of another line is non-zero; (ii) while a line represents an increasing relationship, the other one represents a decreasing relationship; and (iii) both are linear relationships (see Fig. 3).

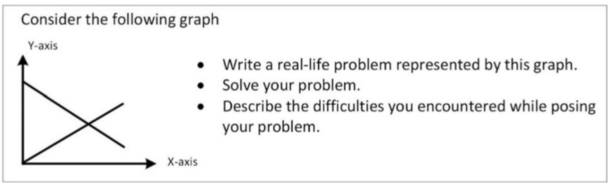
The graph in the task was not situated in a real-world context. However, the task includes the real-world requirement because it asks participants to add a real-world context while posing problems. In particular, the task asks participants to pose a real-world problem that can be represented by the graph. By including this real-life requirement, PMTs' problems could be examined to identify which mathematical information they could identify and interpret in context. Silber and Cai (2021) provide two sample responses to show the role of a real-life requirement in the problem-posing task that includes a linear graph with a non-zero y-intercept and an increasing relationship: "(1) What is the slope of the line? (2) A company got a loan of $2,000 to help start their company. If the company makes $500 a month, how much money have they made by the sixth month?" (p. 881). In the first response, it is not definite whether the problem poser identifies the slope value in the graph without asking the answer to the problem. However, the second response includes a non-zero y-intercept (a starting value of $2000) and an increasing relationship (an increase of $500 a month). Moreover, the second response contains what the dependent and independent variables represent. Therefore, since the second response is contextualized, it is possible to understand how the problem poser identified key mathematical ideas on linear graphs. In such problems, it can be possible to code mathematical ideas invoked by the learners in their posed problems.

According to Possamai and Allevato (2024), "for objectives related to learning certain mathematical knowledge, the prompt must contain more specific requirements" (p.3). In this sense, the task (see Fig. 1) included three prompts that let posers know what they are expected to do. The first prompt asked PMTs to pose a real-life problem represented by the given graph. The second prompt asked PMTs to solve their posed problems. This allows for comparing PMTs' solutions and coders' solutions to the problems. The third prompt asked PMTs to describe the difficulties they encountered while posing problems. This prompt was useful to analyze PMTs' awareness about their own difficulties they encountered in posing problems.

Data collection and analysis

Before starting data collection, because PMTs did not take any course about problem posing in the learning and teaching of mathematics, they were informed about three types of problem-posing tasks through various examples according to Stoyanova and Ellerton's (1996) categorization. The participants received the problem-posing task after a week. The task required individual responses from them within a week. In the task, they were to (1) pose a problem in a real-life context that can be represented by the graph, (2) solve the posed problem, and (3) write about the difficulties they encountered during the problem-posing process. Since it is

Fig. 3 The problem-posing task



difficult to pose problems in the “here and now” (Kontorovich, 2020; Yazgan & Ülker, 2023) and the problems they create may be of low quality, PMTs were asked to send their written responses via email within one week. After an initial analysis of the posed problems, some were based on mathematical and contextual aspects (e.g., contextual and linguistic characteristics of problems and key mathematical ideas related to linear functions). These problems were shared via a graphics tablet on the screen and discussed with PMTs in the classroom to enrich the data. During the class discussion, PMTs were asked to explain how they posed their problems (e.g., How and why did you pose this problem? How did you assign variables? Could you please solve your posed problem again?, etc.). Therefore, PMTs’ detailed written reports, the solutions to their own problems, and classroom discussion data were used to understand and provide evidence for the significant features of the posed problems. The classroom discussion lasted 50 min and was recorded with a camera. Class discussion data was not used in this study because it was not needed. Besides, by using the member-checking method, some participants were asked for additional questions by mail at the end of the research.

In order to analyze the data, the posed problems were labeled P1, P2, …, and P47 in an Excel document. In this study, an adapted version of Cankoy and Özder’s (2017) rubric was used to assess PMTs’ problems in terms of *solvability*, *context*, and *language*. Nevertheless, modifications were made to the scoring rubric due to the distinct nature of the tasks presented, which differed from those utilized in Cankoy and Özder’s (2017) study. For example, this study did not use the mathematical structure dimension to assess problems because of the nature of the graph task. In the literature, learners pose problems about graphs in mathematical or real-world settings (Leinhardt et al., 1990; Nedaei et al., 2022). Moreover, the subcategories of the context dimension in Cankoy and Özder’s (2017) study (routine and non-routine) were revised. In Cankoy and Özder’s (2017) study, since they collected the data from middle school students, they examined the context as routine and non-routine in relation to whether the context of the problems was different from the context used in the classroom and the textbooks. However, since the participants in this study were PMTs, it made more sense to look at whether a problem was realistic rather than routine (Nedaei et al., 2022). The context dimension was divided into two subgroups: mathematical and real-world context. Then the dimension of realism was divided into two distinct subcategories, namely realistic and unrealistic. In addition, *the reasonability* dimension in Cankoy and Özder’s (2017) study is related to “the information given in the problem and the solution is reasonable and applicable in real life” (p. 2428). This study did not use the reasonability dimension to evaluate the problems, as the realism component satisfied this condition. To examine content-specific elements in problems, the dimensions from Silber and Cai’s (2021) and Cai et al.’s (2013) studies were utilized. For instance, covariance, linearity, and y-intercept were employed due to their similarity to the graphical tasks in those studies.

Coding of general-mathematical elements in posed problems

Similar to Silber and Cai’s (2021) study, each posed problem was initially analyzed to determine if the response constituted a mathematical question or not (see Fig. 4). A response was classified as a mathematical question if its solution can be derived using mathematical or quantitative reasoning (see P38 in Table 3). Instances when the response did not take the form of a question but showed signs of employing mathematical or quantitative reasoning were classified as mathematical statements (e.g., *while the distance traveled by a moving vehicle increases, its gasoline consumption*

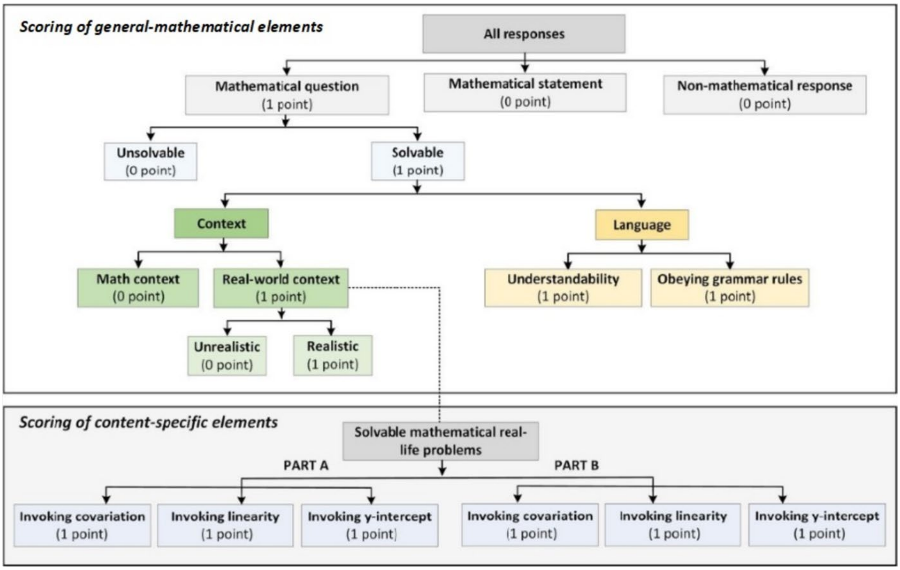


Fig. 4 Scoring of general-mathematical and content-specific elements in posed problems *Note.* General-mathematical elements were adapted from studies on problem posing (Cankoy & Özder, 2017; Nedaie et al., 2022, Silber & Cai, 2021). Content-specific elements were adapted from studies on problem posing on linear graphs (Cai et al., 2013; Silber & Cai, 2021)

decreases over time (y-quantity and x-time)). Responses that were neither mathematical questions nor mathematical statements were coded as non-mathematical responses (e.g., *the rates of speed of two cars and the distances covered. Rate (y) and distance (x)*).

If the posed problem was a mathematical question, one point was assigned to it. Otherwise, it took a zero point (see Fig. 3). If a response was a mathematical question, its solvability was assessed. If a problem was solvable, one point was scored. If the information in the problem is not enough to find a solution, it was coded “not solvable,” and a zero point was allocated to it (see Fig. 4). For the context dimension, similar to Nedaie et al.’s (2022) study, each solvable problem was examined to see whether it was set in a mathematical or real-world context. If a solvable problem was set in a real-world context, it earned one point. The realism dimension was then divided into two subcategories: *realistic* and *unrealistic,* because some posed problems in a real-world context but not in a realistic situation. For *realism*, if a solvable problem was presented in a real-world context, and it was realistic, an additional point was awarded.

The graph task requires PMTs to use mathematical and ordinary language due to the real-life requirement. For this reason, language use becomes a critical component in assessing problems (English, 1998; Van Harpen & Presmeg, 2013). If a posed problem in a real-world context was solvable and the language used in the problem was clear, understandable, and fluent when the coder read the problem, the response earned one point. Moreover, the posed problem obeys grammar rules, so the response earned another point. Accordingly, a problem gets a maximum of 6 points regarding general-mathematical elements.

Table 3 Scoring examples of posed problems based on general-mathematical and content-specific elements

Sample responses	General-mathematical elements						Content-specific elements					
	MQ	S	RWC	R	Language		Part A			Part B		
					Und	Gra	Cov	Lin	Y-int	Cov	Lin	Y-int
P17: Car A is traveling from city X to city Y. Car B goes from city Y to city X on the same road. The distance between city X and city Y is 400 km. The cars set off in the opposite direction at the same time. If car A travels all the way at 110 km/h and car B at 90 km/h, after how many hours will they meet?	1	1	1	1	1	1	0	0	0	0	0	0
P39: On the left side of a scale, there are 10 identical balls weighing 100 g. On the right side, there are 4 balls of 5 g each. How many balls should you remove from the left side of the scale to achieve balance?	1	1	1	1	1	1	0	0	0	0	0	0
P38: There are two pools of equal volume. One of the pools is completely filled with 120 L of water, and the other is empty. We transfer the water from the full pool to the empty one. The pool discharges 6 L of water per minute. Accordingly, what will be the ratio of the volumes of water in the two pools at the end of the 14th minute?	1	1	1	1	1	1	1	1	1	1	1	1
P3: There are two cars on a road. One of the cars is in motion, slowing down at a specific speed. The other has just started to move and is accelerating. Draw a time (x)-speed (y) graph for two cars, according to the story	1	1	1	1	0	1	1	0	1	1	0	1
P36: Workers decide to quit because a factory pays its workers poorly. Workers decide to quit because a factory pays its employees poorly. The factory experiences an accumulation of work as the number of workers decreases, the factory accumulates more work. Draw the changes in the number of workers and the amount of work in accordance with this expression	1	1	1	1	0	0	1	0	0	1	0	0
P10: Leyla runs every morning. If Leyla burns 200 cal in every half hour in the morning run, how many calories will she burn in 2.5 h?	1	1	1	1	1	1	0	0	0	1	1	1
P26: While watching the news, Emre heard these sentences: “Purchasing power decreases as inflation increases” and “Purchasing power increases as inflation decreases”. Show these sentences on a graph. How should that graph be? (x-purchasing power, y-inflation)	1	1	1	0	0	1	1	0	0	0	0	0

MQ, mean mathematical question; S, solvability; RWC, real-world context; R, realism; Und, understandability; Gra, grammar; Cov, covariation; Lin, linearity; Y-int, y-intercept

Coding of content-specific elements in posed problems

For the analysis of content-specific elements in the posed problems, literature related to linear functions (e.g., Leinhardt et al., 1990; Pierce et al., 2010) was examined. The major mathematical ideas were identified based on the studies on problem posing about linear graphs (Cai et al., 2013; Silber & Cai, 2021), such as reflecting (i) *covariation*, (ii) *linearity*, and (iii) the *y-intercept*. These elements were examined in solvable mathematical problems that were set in a real-world context (see Fig. 4). More specifically, since the graph task includes two linear relationships, the major mathematical elements in the posed problems were coded separately for each linear relationship in Part A and Part B. In Part A, it was examined if a posed problem invokes (1) the decreasing nature of the relationship (covariation), (2) linearity, and (3) a non-zero y-intercept. Similarly, in Part B, it was examined if a posed problem invokes (1) the increasing nature of the relationship (covariation), (2) linearity, and (3) a zero y-intercept. These mathematical ideas were used as evidence that the participant had identified mathematical information in the posing situation. A posed problem earned 1 point for each correctly reflected mathematical idea. Thus, a problem reflecting all mathematical ideas correctly earned a maximum of 6 points in the coding of content-specific elements (see examples in Table 3).

The author and an expert independently coded the complete data in terms of both general-mathematical and content-specific elements. There was an inter-rater reliability of 100% for the general-mathematical elements according to Miles and Huberman's (1994) formula. In the data analysis of the content-specific elements, the inter-rater reliability was found to be 88%. Disagreements in coding were discussed, and an agreement was reached on how to code these items. In this study, PMTs' verbal explanations in the classroom discussion were used to support written data. Moreover, PMTs' written solutions to their posed problems were utilized as evidence to indicate how they invoked key mathematical ideas related to the graphs of linear functions. The SPSS package program performed the statistical analyses on the data. For the normality test, skewness and kurtosis values were examined. It was observed that the skewness value was -0.07 and the kurtosis value was -1.21. When kurtosis and skewness values are between -1.5 and + 1.5, it is considered a normal distribution (Tabachnick & Fidell, 2013). Moreover, the data doesn't contain outliers. The sample size was greater than 30; hence, if the sample data is reasonably symmetric, the statistic may distribute approximately normally. For the third research question, a paired-sample t-test was conducted to compare the mean scores for general-mathematical and content-specific elements in the posed problems.

Coding examples related to the quality of problems

Table 3 displays the problem scoring according to general-mathematical and content-specific elements. Two problems were explained in detail to demonstrate the coding process. In P10 (see Table 3), "How many calories will she burn in 2.5 h?" would be considered a mathematical question; the solution requires quantitative reasoning with "the hour Leyla ran" and "the calories she burned." In terms of solvability, there is sufficient information to find a solution to the problem. For instance, the problem's question qualifies as solvable because it provides the number of calories burned every half hour. The problem was based on a real-life scenario involving a running activity. Moreover, the problem is linguistically understandable and grammatically correct. Hence, in terms of general-mathematical elements, the problem gained six points. P10 included only one linear relationship, rather than two linear relationships. In the problem, “the hour Leyla ran” and “the calories she burned” indicate a covariational relationship. “200 cal in every half hour” shows a linear relationship. The problem also shows that the starting point is zero. For this reason, the problem involves a zero y-intercept. However, the problem did not involve a linear relationship. For this reason, P10 gained three points for content-specific elements.

Another example is P26 (see Table 3), which shows data coding. In P26, “How should that graph be?” is considered a mathematical question because the solution requires showing the change in purchasing power according to inflation. This problem is solvable because a problem solver can draw different graphs to show the following relationship: “Purchasing power decreases as inflation increases.” The problem provided a mathematical application in the real world, but not a realistic situation. In a real-world situation, the relationship between these two variables is often too complex for a simple linear equation to explain, and it depends on numerous parameters. In terms of language use, the problem is grammatically correct, but it is not understandable because the nature of the relationship between variables is not definite. As a result, the problem gained four points for general-mathematical elements. In terms of content-specific elements, P26 includes only a covariational relationship. However, it is unclear whether this relationship is linear or not. Furthermore, there is no expression in the problem to reflect the y-intercept value. Hence, this problem gained only one point for content-specific elements.

Coding of PMTs' declarations about their difficulties in posing problems

PMTs’ declarations about their difficulties in posing problems were examined through qualitative content analysis (Hsieh & Shannon, 2005). Both deductive and inductive approaches were utilized in the analysis of data. In the inductive approach, themes and categories arise from the data through the researcher’s meticulous analysis and continuous comparison. In the deductive approach, initial coding is based on a theory or pertinent research findings. To determine initial codes, previous studies regarding teachers’ difficulties in problem posing were examined. In the literature, the following difficulties were frequently cited: lack of experience (Leavy & Hourigan, 2022b; Şengül & Katrancı, 2015), weakness in subject matter knowledge (SMK) and pedagogical content knowledge (PCK) (Leavy & Hourigan, 2022b; Şengül & Katrancı, 2015), difficulty in arranging problems according to students’ levels and school curricula at a specific educational level (Mallart et al., 2018; Şengül & Katrancı, 2015), and difficulty in task-related factors such as whether the task includes numbers or asks a real-life context requirement or contains prompts associated with cognitive demand level (Mallart et al., 2018; Stickles, 2011; Ulusoy, 2023; Ulusoy & Kepceoğlu, 2018). After deductive analysis, the researcher and a mathematics educator, who specializes in problem posing, scrutinized 47 PMTs’ written responses to the task question: “*Describe what difficulties you encountered while posing your problem.*” In this inductive analysis, two coders read all responses multiple times, identifying themes throughout. During the initial reading, we identified five preliminary themes. Some of these themes had a connection to the task, while others were associated with the problem posers’ experience, SMK, and PCK that were similar to the themes in the literature. Following a discussion, we grouped these themes into two overarching categories: *difficulties related to task-related components* and *difficulties related to teacher-related components* (refer to Table 6). Task-related components were divided into two groups as *prompts* and *situations,* considering task variables in problem-posing task in recent studies (e.g., Cai et al., 2023;

Leavy & Hourigan, 2024). Two raters independently coded the PMTs' responses within an Excel document for each theme, assigning a code of 1 if the PMTs mentioned it in their response and 0 if not. The raters were in agreement on 71 of the difficulty-coding decisions (91%). To reach a consensus, two coders discussed the non-matching responses and finalized the codes.

Ethics and research clearance

The current study sought ethical approval from the Provincial Directorate for National Education. Furthermore, it is important to note that the study did not raise any ethical concerns in relation to PMTs' involvement. This is because all participants were required to provide informed consent by signing a consent form, which outlined their agreement to have their reflection papers and classroom discussions videotaped. The name of PMTs also remained anonymous.

Findings

Assessing general-mathematical elements in posed problems

Of forty-seven responses, forty-six were mathematical questions and one was a mathematical statement (e.g., *while the distance traveled by a moving vehicle increases, its gasoline decreases over time (y-quantity and x-time)*. No response fell into the non-mathematical category. Table 4 shows the frequencies of the problems posed regarding solvability, context, and language dimensions. The following sub-headings provided detailed explanations for each dimension.

Solvability

According to Table 4, while 43 (93%) responses were solvable mathematical questions, 3 (7%) responses were mathematical questions but not solvable. For example, in Table 3, P10 was also a solvable problem because the answer can be easily found: 0.5 h → 200 cal, 2.5 h → 1000 cal. However, "P47: *According to the velocity–time graph of the two vehicles, when do the vehicles meet?"* was not a solvable problem. This response would be considered mathematical but not solvable, as the velocity–time graph has no information about when they meet. The PMT's solution showed that he focused on the intersection point on the graph as the meeting point of the vehicles. However, the

Table 4 Analyzing PMTs' mathematical questions (N = 46)

Dimension		Sub-dimensions		Frequency
Solvable	Context	Math context		1
		Real-world context	Realistic	36
			Unrealistic	6
	Language	Clear		19
		Unclear		24
Unsolvable				3

intersection point on the graph does not show the time when the vehicles met. Instead, it shows when their speeds were the same. Therefore, the information given was not enough to reach a response.

Context

Out of 47 problems, only one consisted of variables in an abstract mathematical context. In this sense, PMT13 posed the following problem: *Construct the graphs of* $y=2\times$ *and* $3x+2y=24$ *and find the apsis of the point where the lines intersect*. According to PMT13's solution, variables in the problem were abstract, numerical, and continuous as real numbers. Within the solvable problems set in a real-world context, 36 (86%) were realistic, and six (14%) were unrealistic. For example, P4 and P10 in Table 3 include realistic contexts. In particular, the calories burned values and times in P10 were compatible with real-life. Unrealistic situations were found in problems situated in scientific contexts (e.g., physics, biology, chemistry, and economy). For example, in Table 3, P26 included an unrealistic situation. Figure 5 represents the problem poser's solution to her own problem. This solution assumes a linear relationship between inflation and purchasing power [parity]. In the solution, what the lines represent is not definite. Furthermore, a simple linear equation typically cannot explain the complex relationship between these two variables, which relies on numerous parameters in a real-world scenario.

Another posed problem (P8 in Table 3) states that in a plant experiment, a plant becomes shorter in the dark or when it doesn't receive water. The PMT responded to the question of how they shortened the plant by saying, "*I wanted to pose an original problem. Therefore, I selected a subject that was slightly less mathematical in nature. I thought of plant experiments. I mentioned that the absence of sunlight shortened the height of the wheat grains.* According to these explanations, the PMT stated that the wheat grains grow shorter in a dark and waterless environment, which is incompatible with real life. Moreover, three problems included information inconsistent with a person's walking speed in real life (e.g., *in a walking tour, a child travels 20 m per second* or *a man travels 40 m per second*). The world's best short-distance runner, Usain Bolt, ran 100 m in about 10 s at the 2009 World Championship and broke his world record. That is, it is inconsistent with real-life to set a case where a runner runs 10 m per second while a child travels 20 m per second.

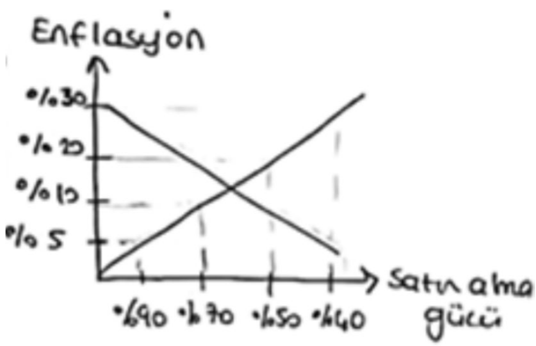


Fig. 5 PMT's solution to P26; *x*-axis: purchasing power, *y*-axis: Inflation

Language

According to Table 4, the language used in 19 (44%) of the solvable problems was clear and understandable. The language of the problems was mostly grammatically correct, but most of them were not clear in terms of understandability. The detailed analysis indicated that the problems where it is not necessary to verbalize key mathematical ideas related to the graphs of linear graphs were mostly linguistically precise (e.g., P17 and P39 in Table 3). These problems primarily require the creation of operations, rather than verbalizing linear relationships between two variables. These problems had less linguistic complexity than verbalizing covariation, linearity, and y-intercepts because of the limited use of mathematical vocabulary. However, the problems that verbalized key mathematical concepts included topic-specific and general language deficiencies. For example, in Table 3, P36 and P3 are required to construct a qualitative graph to show two situations based on the changes in quantity over time. The problem's unclear explanation, which lacks clarity in the rates of decrease and increase and the units of work quantity, allows for a variety of graph drawings to solve these problems. Although we can find different graphs for P36 and P3, none of the solutions represent the graph in the problem-posing task because of the unclear verbalization of key mathematical concepts. This leads to many topic-specific language deficiencies. Mathematical vocabulary becomes highly specialized and technical in those problems since it requires invoking all key mathematical ideas related to the graphs of linear functions in real-world contexts. Interestingly, while PMTs' own solutions to their problems included linearity, their statements lacked academic language about the constant rate of change in a real-life context.

Analysis of content-specific elements in posed problems

Table 5 illustrates how the posed problems reflect key mathematical ideas related to the graph parts (Part A and Part B). According to Table 5, the information about linearity was least accessed, and the idea of covariation between two quantities (in an increasing or decreasing relationship) seems to have been the most accessible.

Figure 6 shows the number of key mathematical elements reflected in the posed problems. The difference between frequencies for Part A and Part B resulted from the fact that some problems contained only Part A of the given graph. For example, in Table 3, P10 reflected the covariation of two quantities in only one scenario. However, the graph task includes two different situations. Only six problems included all the mathematical elements. For instance, P38 in Table 3 illustrates two scenarios with two pools: the covariation of two quantities (transferring water from the full pool to the empty pool), linearity (6 L of water per minute), a non-zero y-intercept (one pool completely filled with 120 L of water), and a zero intercept (the other pool is empty). As a result, this problem invoked all mathematical elements for both parts of the graph task.

The problems included two mathematical elements, which mostly reflected the covariation of two quantities and the y-intercept. For example, P3 in Table 3 reflects two situations (*two cars*), the covariation of two quantities (*slowing down* and *accelerating*), a zero intercept (*just started*), and a non-zero intercept (*a certain speed*). Thus, P3 invoked two mathematical elements for both parts of the graph task. Some problems could reflect only one mathematical element. These problems only reflected the idea of covariation between two quantities (e.g., P3 in Table 3, *the number of workers in the factory decreases*, and *the*

Table 5 Key mathematical concepts in solvable problems in a real-world context (n = 42)

	Part A			Part B		
	Covariation (decreasing)	linearity	non-zero y-intercept	Covariation (increasing)	linearity	zero y-intercept
Frequency	25 (60%)	8 (19%)	19 (45%)	23 (55%)	8 (19%)	17 (40%)

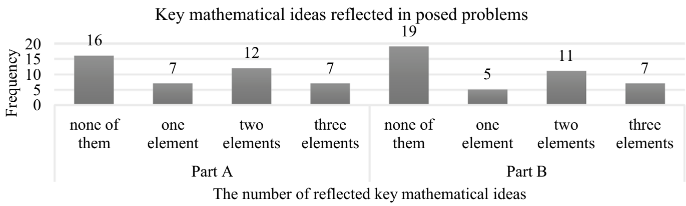


Fig. 6 Reflection of the key mathematical elements in posed problems

work to be done accumulates). Those problems included *decreasing*, *decelerating*, *slowing down*, *increasing*, *accelerating*, and *becoming faster*). These terms show that PMTs focused on increasing and decreasing relationships between variables while posing their problems. Although the problems posed did not invoke linearity, PMTs' solutions for their own problems included straight lines. Despite the necessity of posing a linearity-related problem in the task, the posed problems failed to appropriately express linearity in a real-life context. Finally, almost half of the 42 problems do not contain any of the three elements. P17 and P39 were typical examples of this type of problem (see Table 3). In this type of problem, there was a focus on two situations (e.g., *car A and car B, or the left and right sides of a scale*). However, it could not reflect the covariation of two quantities. Therefore, the capacity to invoke mathematical ideas in the graph task seems to be difficult for problem posers. The results suggest that posing problems with real-life associations (e.g., object, motion, speed, etc.) was demanding since it requires building stronger links between mathematical and relevant contextual knowledge.

The difference between general-mathematical and content-specific elements in the problems

A paired-sample t-test was also conducted to compare the mean scores for general-mathematical and content-specific elements in the posed problems. The results indicated that the mean score for general-mathematical elements in posed problems ($M=5$, $SD=1.1$) was significantly higher than the mean score for content-specific elements in posed problems ($M=2.3$, $SD=2.2$), $t(45)=7.2$, $p<0.001$. Furthermore, the qualitative analysis of the proposed problems supported this conclusion. Figure 7 shows the frequency of problems based on the number of general-mathematical and content-specific

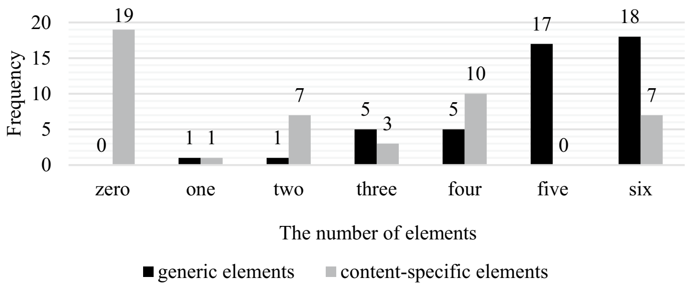


Fig. 7 The frequency of the problems based on general-mathematical and content-specific elements

elements they contain. According to Fig. 7, there are 18 problems that contain all general-mathematical elements, but only 7 problems contain all content-specific elements. For example, although P17 and P39 (see Table 3) had six points based on general-mathematical elements, they had zero points on content-specific elements because they did not reflect any key mathematical ideas presented in the problem-posing task. Additionally, in all the data, only one problem included both general-mathematical and content-specific elements and received a full score of 12 (see P38 in Table 3).

PMTs' declarations about their difficulties in posing problems about linear graphs

Table 6 shows several difficulties that PMTs declared when they posed problems involving linear graphs. The difficulties PMTs described were grouped in two overarching categories: difficulties related to task-related components and difficulties related to teacher-related components.

While posing their problems, almost half of the PMTs cited *task-related components* as a difficulty. Specifically, they mostly provided explanations about the *situation* and/or *prompts* presented in the task. Regarding *the situation*, many PMTs expressed that two linear situations in the task, one increasing and the other decreasing, made it challenging to formulate a problem using a single graph. In particular, PMT45 explicitly stated, "I could construct the problem more easily if there was only one linear function or if both lines were increasing or decreasing in the task." In addition, some PMTs stated that the qualitative characteristics of the graph (i.e., not containing any numerical expression and the uncertainty of the variables) caused difficulty. For example, PMT1 said, "This kind of problem is more difficult to pose and solve than other problems because it is necessary to consider multiple unknowns together in graphical problems (i.e., considering the relationships between variables in a real-life context)." These written statements revealed that they described their difficulties associated with invoking key mathematical elements related to the concept of linear functions while engaging in problem posing. For example, PMT39 wrote, "I had difficulty choosing [a real-life] context because two different lines intersect at a single point. I also had difficulty posing a problem suitable for the graph because one of the two lines decreases and the other increases." Below is an excerpt from the class discussion that explains how they assigned the context of variables to their problems.

Researcher:	How did you select the variables?
PMT3:	Looking at these graphs brought back memories of the physics problems I had studied. I thought of *speed* and *time* as [variables]

Table 6 PMTs' declarations about their difficulties in problem posing

	Task-related components		Teacher-related components*		
	Situation	Prompt	Lack of experience	Weakness in SMK	Weakness in PCK
Frequency (%)	23 (49)	22 (47)	8 (17)	15 (32)	9 (19)

*SMK and PCK refer to *subject matter knowledge* and *pedagogical content knowledge*, respectively

Researcher:	How did you select the variables?
PMT5:	I agree. As soon as I saw the graph, I said this could be a physics problem rather than a mathematics problem
PT13:	In biology, I considered the *osmotic* and *turgor* pressures. I liked this topic very much when I was in high school
Researcher:	Why did you always take *time* for the variable on the *x*-axis?
PMT3:	We are very used to time being a variable
PMT16:	It seemed easier to me. For example, I used *time* and *displacement*

The class discussion revealed that PMTs tended to use variables that they were familiar with from previous learning. Furthermore, they preferred contexts where variables were easy to assign. *Time* was considered relatively easy since things will change over time naturally in real life and do not need special attention due to their unidirectional nature (time only increases). In addition, PMTs identified most frequently the following as continuous dependent variables: *distance* (e.g., length, displacement, and height), *speed*, and *volume*. The literature on graphing and functions also frequently uses these continuous variables. Furthermore, PMTs' educational background indicated that they were familiar with the variables in kinematics contexts due to Physic I-II courses.

Regarding task-related difficulties, 47% of the PMTs described the prompts as a challenge. They frequently noted the following prompt: "*Write a real-life problem represented by this graph*". According to PMTs' written explanations, they struggled to find a suitable context for their problem because of the real-life requirement. They felt that this requirement required verbalizing all unknowns (e.g., variables, slopes, and y-intercepts) in a real-life scenario. Their written explanations indicated that selecting real-life contexts that are relevant and engaging for students while also aligning with linear graphs was challenging. For instance, PMT31 expressed her concern as follows: "I could only think of variables such as speed-time. I couldn't decide which subject in mathematics would be suitable for these graphs. I could only think of direct and inverse proportions in mathematics." Similar to PMT31's explanations, five PMTs said that they only remembered real-life contexts limited to physics, such as velocity, acceleration, and location. Moreover, the prompts specifically requested a real-life problem, represented by the given graph. This prompt made it difficult to pose problems. PMT5 wrote, "I had difficulty making my data suitable for the graph because I tried to ensure that the answer to my problem was the given graph."

The most frequently cited difficulty concerning *teacher-related components was a weakness in SMK regarding linear graphs.* PMT26 explicitly wrote, "I had difficulty because I have poor skills about reading and solving graphs." Moreover, PMT16 said, "I had difficulties posing problems because I realized that I had problems reading and interpreting graphs. I had difficulties remembering the speed-time graphs." PMT16's explanations showed that he associated the graphs only with the contexts he learned in physics courses. This can be considered an indication that some PMTs were insufficient in terms of associating linear graphs with real-life situations that are different from physics contexts. These explanations revealed that it's challenging to pose problems that effectively target specific learning objectives without deep subject matter knowledge regarding linear graphs. In addition, 17% of the PMTs declared their lack of experience in posing problems about graphical situations as a difficulty because they were more accustomed to problem-posing activities that included numerical expressions and contextualized information. For example, PMT13 wrote, "I have never had any experience of posing problems with graphs before." PMT11 said, "I actually took a problem-posing course, but I did not attend it because there was no attendance obligation. I had difficulties with problem posing due to a lack of experience. Additionally, our high school education involved teacher-centered lessons. I am more comfortable solving problems than posing them." These explanations revealed that, despite his limited experience with problem posing, he attributed his difficulty to traditional, teacher-led instruction where the emphasis was on problem solving rather than problem posing. This lack of exposure can result in PMTs feeling ill-equipped to design open-ended problem-posing activities.

According to Table 6, 19% of the PMTs declared their weakness in pedagogical content knowledge as a difficulty. According to these PMTs, posing problems that align with students' cognitive level is a challenging process. For example, PMT22 wrote, "The problem I posed was too long due to a real-life requirement, and it may be difficult for students to read and understand." This explanation showed that although this PMT predicted whether the student would find the problem easy or difficult, she explained that she had difficulty adjusting the reading complexity of the problem according to students' cognitive level. As another example, PMT27 stated, "I struggled to formulate problems that align with middle school students' knowledge about linear graphs. The problem I presented requires students to understand speed-time relationships. I am uncertain whether they have grasped these psychic concepts or not." These explanations revealed that PMTs felt unsure how to understand students' knowledge in mathematics and psychic. They also expressed concerns about whether middle school mathematics and science curricula covered the contexts they utilized.

Discussion and conclusions

This study aims to assess the quality of PMTs' problems in a problem-posing context related to linear graphs through the analysis of general-mathematical and content-specific elements. In this study, an adapted version of Cankoy and Özder's (2017) rubric was used to assess PMTs' problems in terms of *solvability*, *context*, *realism*, and *language*. Hence, the adapted version of the rubric allows a deeper analysis of PMTs' posed problems from different perspectives. For the analysis of key mathematical elements in posed problems, the dimensions in Silber and Cai's (2021) and Cai et al.'s (2013) studies were used (e.g., invoking *covariation*, *linearity*, and *y-intercept).* Before PMTs incorporate problem posing into their practice, it is significant to assess and increase their own problem posing performance (Cai & Hwang, 2020; Cai et al., 2020). However, Cai et al. (2023) assert that, despite possessing well-defined criteria for evaluating problem-solving success, the quality of the posed problems is often ambiguous. The present study revealed that the analysis of general-mathematical elements is not sufficient to determine the quality of posed problems because posing problems in particular subject matters imposes additional content-specific entailments. The results of this study revealed that integrating general-mathematical and content-specific elements in assessing the quality of problems provided a better understanding of PMTs' problem-posing performances. In this regard, the assessment of the quality of posed problems through general-mathematical and content-specific elements provides opportunities for the researchers to devise specific tactics for handling the learners' posed problems in particular. Additionally, it can serve as a valuable instrument for developing curricula and instructional materials pertaining to linear graphs.

The first research question focused on the quality of posed problems in terms of general-mathematical elements (solvability, context, and language). The results showed that the posed problems were mostly solvable. The results concerning solvability and context align with previous findings of many studies (e.g., Cai et al., 2020; Nedaei et al., 2022). Moreover, PMTs were successful in using a real-life context to solve their problems. Similarly, Işık et al. (2012) observed that the majority of prospective primary school teachers are paying attention to providing contextual content. However, Cai et al. (2013) stated that only 58% of the 338 problems posed by high school (11th grade) students are contextual. The differences between the levels of PMTs' and students' mathematical and contextual knowledge may explain the similarities and differences in the study results. On the other hand, the use of language was clear and understandable in 44% of solvable problems. However, most of these problems did not respond to the problem-posing task, even though they were linguistically precise. This finding did not support previous problem-posing studies (Cankoy & Özder, 2017; Nedaei et al., 2022; Silver & Cai, 1996) that have reported the majority of posed problems were understandable. In general, the problems with topic-specific language deficiencies could not reflect linearity. The posed problems included the terms such as increasing/accelerating or decreasing/decelerating without providing an explanation of the changes in one variable in relation to another. This finding aligns with studies that highlight the primary challenge of reflecting linearity in a problem-posing context (Cai et al., 2013; Kar, 2016; Silber & Cai, 2021). One possible reason could be the absence of a word in Turkish that accurately describes the rate of change (Ulusoy, 2020; Yemen-Karpuzcu, Ulusoy, & Işıksal-Bostan, 2017). Moreover, according to PMTs' written explanations about their difficulties in posing problems, their imprecise language use in the problems can be related to their previous experiences with the tasks associated with linear graphs. The textbooks and exam questions mostly require interpreting given graphs or plotting points from a table of ordered pairs and connecting them in a straight line in a mathematical context (İncikabı & Biber, 2018; Leinhardt et al., 1990). Such tasks do not necessitate learning the mathematical vocabulary associated with linear graphs in real-life situations. The limitations in PMTs' specialized content area language may create a barrier to enhancing student comprehension of content (Livers & Elmore, 2018; Rubenstein & Thompson, 2002). The current study supports the idea that it is necessary to develop PMTs' use of mathematical language in problem-posing context. Future studies may provide initial instruction to PMTs to promote the understanding and storage of topic-specific vocabulary (Riccomini et al., 2015) through problem-posing activities that allow active engagement with the language and vocabulary of mathematics (Prediger et al., 2019).

The second research question focused on the quality of posed problems in terms of content-specific elements of linear graphs (covariation, linearity, and y-intercepts). The results indicated that most of the posed problems could reflect only one or two key mathematical elements. In accordance with the studies in the literature (e.g., Hattikudur et al., 2012; Pierce et al., 2010), while the y-intercept can be expressed in most of the problems, linearity could not be fully reflected. The problems contained *increasing/accelerating* or *decreasing/decelerating*, but they failed to explain the nature of changes in a variable with respect to another one. This finding corroborated studies that identified reflecting linearity in a problem-posing context as the primary challenge (Cai et al., 2013; Kar, 2016). For example, Silber and Cai's (2021) study reported that the information about the slope was the least accessed mathematical idea in the graph task. Similarly, Cai et al. (2013) noted that the linear relationship in the task was successfully identified in only 19% of students' problems. One reason may be that there is no word for the term "rate of change" in Turkish (Ulusoy, 2023; Yemen-Karpuzcu et al., 2017). Another reason might be that the textbooks mainly focus on the procedural introduction of slopes and y-intercepts in a graph or equation in abstract settings rather than interpreting and constructing the meaning of these parameters in a real-world context (Arnold & Son, 2011; Davis, 2007). According to Cai et al. (2013) and PMTs' explanations, the other reason could be the complexity of expressing two linear relationships in a single graph. Because of this complexity and the qualitative nature of the problem-posing situation, PMTs may focus on only the sign of the slope of the line rather than verbalizing the constant rate of change correctly (Kar, 2016). In the study, the results confirmed that y-intercept is a simpler object than slope but revealed that it is often omitted from PMTs' verbal descriptions in less than half of the posed problems. In the problems where y-intercept was not omitted, particular terms were used to indicate whether the y-intercept is zero (e.g., empty pool) or not (initial value, starting point, a fixed amount of money at the beginning). One reason can be learners' limited experience with the y-intercept in real-life contexts (Davis, 2007; Hattikudur et al., 2012). Additionally, PMTs can omit the y-intercept in their verbal explanations since the y-intercept seems secondary for them rather than a key feature of a linear function (Pierce et al., 2010).

The third research question focused on the differences between scores on general-mathematical and content-specific elements in the posed problems. When all these elements were quantified, it was found that the mean score for general-mathematical elements in the posed problems was significantly higher than the mean score for content-specific elements in the problems. This result sheds light on the limitations of analyzing the quality elements of the problems by only looking at the general-mathematical elements. Moreover, the results provided evidence that shows general-mathematical and content-specific elements do not appear to represent two totally distinct categories, but rather common points of intersection. For instance, this study primarily linked the use of language elements to the ability to invoke key mathematical ideas. Future studies might examine these complex relations to provide a comprehensive theoretical framework for the assessment of the quality of problems. Even though it is a discrete view of these two components, examining the relationship between them also provides a statistical view of the quality of the problems posed by PMTs. This statistical analysis can help us understand in what kind of situations there is an interaction between these elements. This may be useful to predict the level of reflection of general-mathematical and content-specific elements in problems constructed according to different mathematics topics or task types.

The fourth research question focused on PMTs' declarations associated with their difficulties in posing problems. The results indicated that PMTs described mostly task-related variables (*prompts* and *situations*). This result can be evaluated as the PMTs' awareness of the role of different prompts and situations in learners' problem-posing performance (Baumanns & Rott, 2024; Cai et al., 2023). They particularly focused on the requirement of real-life context as a challenge because it requires verbalization of key mathematical elements in the problems. This result could potentially explain why the posed problems received low scores for reflecting content-specific elements. This result also supports studies where real-life context requirements in problem-posing tasks are frequently cited as a challenge for teachers (Stickles, 2011; Ulusoy, 2023). PMTs also cited their lack of experience in problem posing and partly their weakness in subject matter knowledge and pedagogical content knowledge as the reasons for their difficulties. This result is in line with previous research (Leavy & Hourigan, 2022b; Şengül & Katrancı, 2015). In PMTs' declarations about their difficulties, there is no statement associated with affective-motivational components. This study agrees that there are many factors that affect how teachers pose problems, including teachers' affective and motivational traits, their metacognitive abilities, and their worries and difficulties with problem posing (Leavy & Hourigan, 2022b; Li et al., 2020). In this study, PMTs' problems and explanations were utilized as an evaluative tool to analyze the quality of posed problems. However, future studies can focus on the interactions between problem quality and affective-motivational components.

Limitations and future directions

This study had limitations that may have implications for future research. In this study, PMTs' declarations about their difficulties in problem posing indicated that PMTs may lack a deep understanding of mathematical concepts and connections, which hinders their ability to pose problems. However, this study does not explore difficulties PMTs encountered but the difficulties they declared that they encountered. For this reason, further studies can examine the interactions between teachers' knowledge and problem-posing performances in various problem-posing tasks (Kilic, 2015; Van Harpen & Presmeg, 2013). Such a study allows researchers to understand what the knowledge components are needed to pose a high-quality problem.

The type of task was another limitation because studies on problem posing indicated that the types of graph tasks influence PMTs' problem-posing performance (Kar, 2016; Ulusoy, 2023). For example, Ulusoy (2023) found that PMTs had more difficulty reflecting key mathematical elements in graph tasks where linear changes were different (increasing and decreasing) than in graph tasks where linear changes were similar (both were decreasing or increasing). Moreover, the number of linear relationships in the graph task is another factor that influences learners' problem-posing performance (Kar, 2016). More research can be done to find out how PMTs use both general and specific mathematical elements, such as linear and nonlinear relationships, in their problems when they are working on different graph tasks. In such a study, it might be possible to examine how PMTs distinguish between linear and nonlinear relationships between variables in real-world contexts. Furthermore, exposing PMTs to a variety of graph scenarios in real-life contexts may broaden their perspectives and improve their ability to relate to linear graphs. Thus, it would be possible to gather strong evidence of how different problem-posing tasks or series of problem-posing tasks may foster the acquisition of a conceptual understanding of linear graphs.

The nature of the task's prompts may be a limitation. The prompts in a problem-posing task shape learners' engagement with the task (Cai et al., 2023). In the current study, the prompt asked PMTs to both pose a problem in a real-world context and solve it. Possamai and Allevato (2024) noted that when students encounter a problem-posing prompt with detailed requirements, they tend to consider various aspects of the problem, including its solvability and complexity. In this sense, this prompt might influence the solvability of the problems since 93% of the problems they posed were solvable. Could the results have been different in terms of solvability if PMTs did not solve the problems they posed? Such studies inform educators to understand how different prompts elicit learners' different cognitive processes and problem-posing performance (Silber & Cai, 2017).

Author contribution Fadime Ulusoy helped in validation, writing–original draft, conceptualization, methodology, formal analysis, visualization, supervision, project administration, and data curation.

Funding Open access funding provided by the Scientific and Technological Research Council of Türkiye (TÜBİTAK). This study was supported by Kastamonu University Scientific Research Project Unit (Project no: KÜ-BAP01/2018–27).

Data availability The author does not have permission to share data.

Declarations

Conflict of interests The author declared she has no known competing interest to disclose.

References

Ali, N., & Peebles, D. (2013). The effect of gestalt laws of perceptual organization on the comprehension of three-variable bar and line graphs. *Human Factors, 55*(1), 183–203.

Arnold, L., & Son, J. (2011). Opportunities to conceptualize linear relationships in United States mathematics textbooks: Beyond y=mx+b. In L. R. Weist & T. Lamberg (Eds.), *Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 381–389). University of Nevada.

Bardini, C., & Stacey, K. (2006). Students' conceptions of m and c: How to tune a linear function. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.). *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 113–120). Psychology of Mathematics Education.

Baumanns, L., & Rott, B. (2024). Problem-posing tasks and their influence on pre-service teachers' creative problem-posing performance and self-efficacy. *The Journal of Mathematical Behavior, 73*, Article 101130.

Bayazit, I., & Gray, E. (2006). Acontradiction between pedagogical content knowledge and teaching indications. In J. Novotná, H. Moraová, M. Krátká, & N. Stehliková (Eds.), *Proceedings of 31st international conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 121–128). PME.

Bell, A., & Janvier, C. (1981). The interpretation of graphs representing situations. *For the Learning of Mathematics, 2*(1), 34–42.

Billings, E. M., & Klanderman, D. (2000). Graphical representations of speed: Obstacles preservice K-8 teachers experience. *School Science and Mathematics, 100*(8), 440–450.

Bonotto, C., & Santo, L. D. (2015). On the relationship between problem posing, problem solving, and creativity in the primary school. *Mathematical problem posing: From research to effective practice*, 103–123.

Boote, S. K., & Boote, D. N. (2017). Leaping from discrete to continuous independent variables: Sixth graders' science line graph interpretations. *The Elementary School Journal, 117*(3), 455–484.

Bowen, G. M., & Roth, W. M. (2005). Data and graph interpretation practices among preservice science teachers. *Journal of Research in Science Teaching: THe Official Journal of the National Association for Research in Science Teaching, 42*(10), 1063–1088.

Brown, S. I., & Walter, M. I. (2005). *The art of problem posing*. Psychology Press.

Cai, J., Chen, T., Li, X., Xu, R., Zhang, S., Hu, Y., & Song, N. (2020). Exploring the impact of a problem-posing workshop on elementary school mathematics teachers' conceptions on problem posing and lesson design. *International Journal of Educational Research, 102*, 101404.

Cai, J., & Hwang, S. (2020). Learning to teach through mathematical problem posing: Theoretical considerations, methodology, and directions for future research. *International Journal of Educational Research, 102*, Article 101391.

Cai, J., Hwang, S., Jiang, C., & Silber, S. (2015). Problem-posing research in mathematics education: Some answered and unanswered questions. In J. Cai, N. Ellerton, & F. M. Singer (Eds.), *Mathematical problem posing: From research to effective practice* (pp. 3–34). Springer.

Cai, J., Moyer, J. C., Wang, N., Hwang, S., Nie, B., & Garber, T. (2013). Mathematical problem posing as a measure of curricular effect on students' learning. *Educational Studies in Mathematics, 83*(1), 57–69.

Cai, J., Ran, H., Hwang, S., Ma, Y., Han, J., & Muirhead, F. (2023). Impact of prompts on students' mathematical problem posing. *The Journal of Mathematical Behavior, 72*, Article 101087.

Calabrese, J., Kopparla, M., & Capraro, M. M. (2022). Examining young children's multiplication understanding through problem posing. *Educational Studies, 48*(1), 59–74.

Cañadas, M. C., Molina, M., & Del Rio, A. (2018). Meanings given to algebraic symbolism in problem-posing. *Educational Studies in Mathematics, 98*, 19–37.

Cankoy, O., & Özder, H. (2017). Generalizability theory research on developing a scoring rubric to assess primary school students' problem posing skills. *Eurasia Journal of Mathematics, Science and Technology Education, 13*(6), 2423–2439.

Christou, C., Pitta-Pantazi, D., & Chimoni, M. (2024). Teachers' mathematical problem posing: The role of processes and complexity levels in posing problems on the fraction part-whole concept. *The Journal of Mathematical Behavior, 73*, Article 101134.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*. Author.

Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in preservice teachers' practices. *Educational Studies in Mathematics, 52*, 243–270.

Crespo, S., & Harper, F. K. (2020). Learning to pose collaborative mathematics problems with secondary prospective teachers. *International Journal of Education Research, 102*, Article 101430.

Davis, J. D. (2007). Real-world contexts, multiple representations, student-invented terminology, and y-intercept. *Mathematical Thinking and Learning, 9*(4), 387–418.

English, L. D. (1997). The development of fifth-grade children's problem-posing abilities. *Educational Studies in Mathematics, 34*(3), 183–217.

English, L. D. (1998). Children's problem posing within formal and informal contexts. *Journal for Research in Mathematics Education, 29*(1), 83–106.

English, L. D., & Halford, G. S. (1995). *Mathematics education models and processes*. Laurence Erlbaum.

Fuson, K. C. (1992). Research on whole number addition and subtraction. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 243–275). Macmillan Publishing Company.

Glazer, N. (2011). Challenges with graph interpretation: A review of the literature. *Studies in Science Education, 47*(2), 183–210.

Hartmann, L. M., Schukajlow, S., Niss, M., & Jankvist, U. T. (2024). Preservice teachers' metacognitive process variables in modeling-related problem posing. *The Journal of Mathematical Behavior, 76*, Article 101195.

Hattikudur, S., Prather, R. W., Asquith, P., Alibali, M. W., Knuth, E. J., & Nathan, M. (2012). Constructing graphical representations: Middle schoolers' intuitions and developing knowledge about slope and y-intercept. *School Science and Mathematics, 112*(4), 230–240.

Hsieh, H. F., & Shannon, S. E. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research, 15*(9), 1277–1288.

Huang, R., & Kulm, G. (2012). Prospective middle grade mathematics teachers' knowledge of algebra for teaching. *The Journal of Mathematical Behavior, 31*(4), 417–430.

İncikabı, S., & Biber, A. Ç. (2018). Transitions among the representations in the middle school mathematics textbooks. *Kastamonu Education Journal, 26*(3), 729–740.

Işık, C., & A. Ve Kar, T. (2011). Analysis of the problems related to verbal and visual representations posed by pre-service mathematics teachers. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi, 30*(2), 39–49.

Işık, C., Kar, T., İpek, A. S., & Işık, A. (2012). Difficulties encountered by pre-service classroom teachers in constructing stories about line graphs. *International Online Journal of Educational Sciences, 4*(3), 644–658.

Kalchman, M., & Koedinger, K. R. (2005). Teaching and learning functions. In M. S. Donovan & J. D. Bransford (Eds.), *How students learn: History, mathematics, and science in the classroom* (pp. 351–393). National Academies Press.

Kar, T. (2016). Prospective middle school mathematics teachers' knowledge of linear graphs in context of problem-posing. *International Electronic Journal of Elementary Education, 8*(4), 643–657.

Kerslake, D. (1977). The understanding of graphs. *Mathematics in School, 6*(2), 22–25.

Kilic, C. (2015). Analyzing pre-service primary teachers' fraction knowledge structures through problem posing. *Eurasia Journal of Mathematics, Science and Technology Education, 11*(6), 1603–1619.

Koedinger, K. R., & Nathan, M. J. (2004). The real story behind story problems: Effects of representations on quantitative reasoning. *The Journal of the Learning Sciences, 13*(2), 129–164.

Koichu, B. (2020). Problem posing in the context of teaching for advanced problem solving. *International Journal of Educational Research*, *102*, 101428.
Kontorovich, I. (2020). Problem-posing triggers or where do mathematics competition problems come from? *Educational Studies in Mathematics, 105*(3), 389–406.
Kwek, M. L. (2015). Using problem posing as a formative assessment tool. In F. M. Singer, N. Ellerton, & J. Cai (Eds.), *Mathematical problem posing: From research to effective practice* (pp. 273–292). Springer.
Leavy, A., & Hourigan, M. (2022a). The framework for posing elementary mathematics problems (F-PosE): Supporting teachers to evaluate and select problems for use in elementary mathematics. *Educational Studies in Mathematics, 111*(1), 147–176.
Leavy, A., & Hourigan, M. (2022b). Balancing competing demands: Enhancing the mathematical problem posing skills of prospective teachers through a mathematical letter writing initiative. *Journal of Mathematics Teacher Education, 25*(3), 293–320.
Leavy, A., & Hourigan, M. (2024). Attending to task variables when engaging in group problem posing for elementary level mathematics. *The Journal of Mathematical Behavior, 73*, Article 101128.
Lee, S. Y. (2021). Research status of mathematical problem posing in mathematics education journals. *International Journal of Science and Mathematics Education, 19*(8), 1677–1693.
Lee, Y., Capraro, R. M., & Capraro, M. M. (2018). Mathematics teachers' subject matter knowledge and pedagogical content knowledge in problem posing. *International Electronic Journal of Mathematics Education, 13*(2), 75–90.
Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60*(1), 1–64.
Leung, S. S., & Silver, E. A. (1997). The role of task format, mathematics knowledge, and creative thinking on the arithmetic problem posing of prospective elementary school teachers. *Mathematics Education Research Journal, 9*(1), 5–24.
Li, X., Song, N., Hwang, S., & Cai, J. (2020). Learning to teach mathematics through problem posing: Teachers' beliefs and performance on problem posing. *Educational Studies in Mathematics, 105*, 325–347.
Lin, P. J. (2004). Supporting teachers on designing problem-posing tasks as a tool of assessment to understand students' mathematical learning. In M.J. Hoines & A. B. Fuglestad (Eds.), Paper presented in *proceedings of the 28th conference of the international group for the psychology of mathematics education*, (ss. 3–257–264). Bergen University College.
Livers, S. D., & Elmore, P. (2018). Attending to precision: Vocabulary support in middle school mathematics classrooms. *Reading & Writing Quarterly, 34*(2), 160–173.
Lowrie, T., & Diezmann, C. M. (2009). National numeracy tests: A graphic tells a thousand words. *Australian Journal of Education, 53*(2), 141–158.
Mallart, A., Font, V., & Diez, J. (2018). Case study on mathematics pre-service teachers' difficulties in problem posing. *Eurasia Journal of Mathematics, Science and Technology Education, 14*(4), 1465–1481.
Marco, N., & Palatnik, A. (2024). Teachers pose and design context-based mathematics tasks: What can be learned from product evolution? *Educational Studies in Mathematics, 115*(2), 223–246.
Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Sage.
Ministry of National Education [MoNE] (2018). *Elementary mathematics curriculum.* Milli Eğitim Bakanlığı Yayınları.
Nedaei, M., Radmehr, F., & Drake, M. (2022). Exploring undergraduate engineering students' mathematical problem-posing: The case of integral-area relationships in integral calculus. *Mathematical Thinking and Learning, 24*(2), 149–175.
Patahuddin, S. M., & Lowrie, T. (2019). Examining teachers' knowledge of line graph task: A case of travel task. *International Journal of Science and Mathematics Education, 17*(4), 781–800.
Peden, B. F., & Hausmann, S. E. (2000). Data graphs in introductory and upper level psychology textbooks: A content analysis. *Teaching of Psychology, 27*(2), 93–97.
Peebles, D., & Ali, N. (2015). Expert interpretation of bar and line graphs: The role of graphicacy in reducing the effect of graph format. *Frontiers in Psychology, 6*, 1–11.
Pierce, R., Stacey, K., & Bardini, C. (2010). Linear functions: teaching strategies and students' conceptions associated with y = mx + c. *Pedagogies: an International Journal, 5*(3), 202–215. https://doi.org/10.1080/1554480X.2010.486151
Possamai, J. P., & Allevato, N. S. G. (2024). Teaching mathematics through problem posing: Elements of the task. *The Journal of Mathematical Behavior, 73*, Article 101133.
Prediger, S., Erath, K., & Moser Opitz, E. (2019). Language challenges for students with mathematics difficulties: An overview on research results and instructional approaches. In A. Fritz, V. Haase, & P. Räsänen (Eds.), *International handbook of math learning difficulties: From the laboratory to the classroom* (pp. 437–455). Springer.

Riccomini, P. J., Smith, G. W., Hughes, E. M., & Fries, K. M. (2015). The language of mathematics: The importance of teaching and learning mathematical vocabulary. *Reading & Writing Quarterly, 31*(3), 235–252.

Riley, M. S., & Greeno, J. G. (1988). Developmental analysis of understanding language about quantities and of solving problems. *Cognition and Instruction, 5*(1), 49–101.

Ritter, D., & Coleman, S. L. (1995). Assessing the graphing skills of pre-service elementary teachers: Identifying strenghths and deficiencies in education students' learning processes. *Journal of College Science Teaching, 24*(6), 388–391.

Rubenstein, R. N., & Thompson, D. R. (2002). Understanding and supporting children's mathematical vocabulary development. *Teaching Children Mathematics, 9*(2), 107–112.

Şengül, S., & Katranci, Y. (2015). Free problem posing cases of prospective mathematics teachers: Difficulties and solutions. *Procedia-Social and Behavioral Sciences, 174*, 1983–1990.

Silber, S., & Cai, J. (2017). Pre-service teachers' free and structured mathematical problem posing. *International Journal of Mathematical Education in Science and Technology, 48*(2), 163–184.

Silber, S., & Cai, J. (2021). Exploring underprepared undergraduate students' mathematical problem posing. *ZDM—Mathematics Education, 53*(4), 877–889. https://doi.org/10.1007/s11858-021-01272-z

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19–28.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*(5), 521–539.

Singer, F. M., Ellerton, N., & Cai, J. (2013). Problem-posing research in mathematics education: New questions and directions. *Educational Studies in Mathematics, 83*(1), 1–7.

Stebbins, R. (2001). *Exploratory research in the social sciences*. SAGE Publications, Inc. https://doi.org/10.4135/9781412984249

Stickles, P. R. (2011). An analysis of secondary and middle school teachers' mathematical problem posing. *Investigations in Mathematics Learning, 3*(2), 1–34.

Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students' problem posing in school mathematics. In P. Clarkson (Ed.), *Technology in mathematics education* (pp. 518–525). MERGA.

Stump, S. L. (2001). High school precalculus students' understanding of slope as measure. *School Science and Mathematics, 101*(2), 81–89.

Tabachnick, B. G., & Fidell, L. S. (2013). *Using multivariate statistics*. Pearson.

Ulusoy, F. (2020). Prospective teachers' graphical representations for two simultaneously changing quantities in dynamic functional situations. *Journal of Qualitative Research in Education*, *8*(2). 462-488. https://doi.org/10.14689/issn.2148-624.1.8c.2s.3m

Ulusoy, F. (2023). Middle level mathematics teachers' problems on linear relationships in a semi-structured problem posing situation. *RMLE Online*, *46*(10), 1-19.

Ulusoy, F., & Kepceoğlu, İ. (2018). The contextual and cognitive structure of prospective middle school mathematics teachers' problems in semi-structured problem-posing context. *Journal of Kirsehir Education Faculty*, *19*(3), 1910-1936. https://doi.org/10.1007/s10763-025-10542-1

Van Dooren, W., De Bock, D., Hessels, A., Janssens, D., & Verschaffel, L. (2005). Not everything is proportional: Effects of age and problem type on propensities for overgeneralization. *Cognition and Instruction, 23*(1), 57–86.

Van Harpen, X. Y., & Presmeg, N. C. (2013). An investigation of relationships between students' mathematical problem-posing abilities and their mathematical content knowledge. *Educational Studies in Mathematics, 83*(1), 117–132.

Van Harpen, X. Y., & Sriraman, B. (2013). Creativity and mathematical problem posing: An analysis of high school students' mathematical problem posing in China and the USA. *Educational Studies in Mathematics, 82*(2), 201–221.

Wavering, M. J. (1989). Logical reasoning necessary to make line graphs. *Journal of Research in Science Teaching, 26*(5), 373–379.

Wessman-Enzinger, N. M., & Tobias, J. M. (2022). The dimensions of prospective elementary and middle school teachers' problem posing for integer addition and subtraction. *Journal of Mathematics Teacher Education, 25*, 1–33.

Yazgan, Y., & Ülger, T. K. (2023). Non-routine problem posing and prospective middle school mathematics teachers: An emotional perspective. *International Journal of Mathematical Education in Science and Technology, 54*(10), 1945–1965.

Yemen-Karpuzcu, S., Ulusoy, F., & Işıksal-Bostan, M. (2017). Prospective middle school mathematics teachers' covariational reasoning for interpreting dynamic events during peer interactions. *International Journal of Science and Mathematics Education*, *15*, 89-108.

Zhang, L., Stylianides, A. J., & Stylianides, G. J. (2025). Approaches to supporting and measuring mathematical problem posing: a systematic review of interventions in mathematics education. *International Journal of Science and Mathematics Education*, 1–30. https://doi.org/10.1007/s10763-025-10542-1